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A study of the application of the lognormal distribution to corrective maintenance repair time

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THESIS

A STUDY OF THE APPLICATION OF THE
LOGNORMAL DISTRIBUTION TO
CORRECTIVE MAINTENANCE REPAIR TIME

by

Ronny Almog

June 1979

Thesis Advisor: M. B. Kline

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A Study of the Application of the Lognormal Distribution to Corrective Maintenance Repair Time

by

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The usual mathematical formulation of availability assumes an exponential distribution for failure and repair times. While such an assumption is sometimes correct for reliability, it is not valid for maintainability. This study was conducted primarily in order to verify that the lognormal distribution is a suitable descriptor for corrective maintenance repair times, and to estimate the error caused in assuming an exponential distribution for availability and maintainability calculations when in fact the distribution is lognormal. Approximately 20 sets of existing maintainability demonstration repair time data, of essentially electronic systems, were analyzed using the methods of probability plotting and statistical testing for distributional assumption. The results show that the lognormal distribution assumption cannot be rejected in most of the cases, while the exponential distribution is rejected. However, the error caused when assuming an exponential distribution for MTTR is found to be negligible.

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I. INTRODUCTION

A. OVERVIEW

The effectiveness of a system depends not only on its ability to meet its specified performance requirements, but also on its ability to perform when needed, for the duration of its assigned missions, and for its operational lifetime.

The technical disciplines concerned with these time-related system characteristics are reliability, maintainability and logistics. These are related mathematically by the concepts of availability, dependability and operational readiness.

In order to achieve these performance requirements in the design of a system, it should be possible to translate them into measurable quantitative parameters, and then, to demonstrate their results during the system acceptance testing.

The usual mathematical formulation of availability, when derived from calculus, assumes an exponential distribution for failure and repair times. While such an assumption is sometimes correct for reliability, it is not valid for maintainability since a repair time distribution must start with a value of zero for repair time, and not with its maximum value, at time t=0. No repairs can be made in zero time. In fact, there appears to be overwhelming

evidence that the lognormal distribution is the "best" descriptor for corrective maintenance repair times.

The logarithmic normal density function as a repair time distribution is characterized by a value of zero at time t=0, rises to its maximum value in a reasonably short time, and gradually decreases towards zero as repair times increases [Refs. 1 and 2]. Military standards for prediction and demonstration of maintainability generally are based on the assumption of the lognormal distribution [Ref. 3]. However, field data has shown that repair times are usually longer than specified, predicted, or demonstrated. While part of this can be attributed to differences between the design, the testing, and the field environment, part of it is also due to incorrect assumptions or faulty analytic techniques in the evaluation of the repair time.

In order to help in focusing attention on these matters, a statistical analysis on data sets of demonstrated repair times has been conducted as part of a preliminary study on the application of the lognormal distribution to corrective maintenance downtime. The results of the analysis are given in this thesis.

B. PURPOSE AND APPROACH

1. Objectives

The objectives of this study have been -- (1) to verify that the lognormal distribution is a suitable descriptor for corrective maintenance repair times, (2) to estimate the percentage error caused in assuming an exponential

distribution for availability and maintainability calculations when in fact the distribution is lognormal, (3) to test the lognormal and exponential distributions for systems and equipments in which new technologies in microcircuitry and computation are used to increase reliability and decrease diagnostic time, (4) to determine expected ranges of the principal distribution parameters for different classes of equipment, (5) to test the lognormal and exponential distributions against mechanical and other non-electronic systems.

2. Systems and Data Analyzed

Approximately 20 sets of existing maintainability demonstration repair time data for essentially electronic systems/equipments, were accumulated. Some of the data sets were obtained from published papers and reports. Detailed reports were provided by the Maintainability Assurance Branch of the Engineering Services Division of Ford Aerospace and Communications Corporation.

The systems/equipment analyzed and their sources are discussed in Section IV. They range from 1950's-1960's systems representative of primarily analog, vacuum tube, discrete component design to some 1970's systems using digital, transistor/microelectronics design with extensive built-in test and modular replacement maintainability design features.

The repair times were reviewed for conditions under which taken, accuracy, and specific data points which could cause bias in the analysis.

3. Analysis Approach

Two different approaches to assess the reasonableness of a selected distribution on the basis of given data were considered. These two techniques are (1) probability plotting, and (2) statistical testing for distributional assumptions.

Although these techniques were used primarily to test the assumption of the lognormal distribution for corrective maintenance repair time, the exponential distribution, often assumed in theory, was also tested in order to verify its validity for repair times.

The following procedure was used for analyzing the data:

(a) The data were plotted on lognormal probability paper and the "best fit" line drawn.

(b) A chi-squared goodness-of-fit test [Ref. 4] was performed for the lognormal and the exponential distribution, using a computer program prepared for the analysis.

(c) Another test, due to Shapiro and Wilk [Refs. 4 and 5], called the W-test was used to test the assumption for the lognormal distribution for samples of size less than or equal to 50 (due to availability of tables).

(d) In those cases where the analysis indicated close results for both the lognormal and exponential distribution assumptions, or when the exponential

distribution appeared to be appropriate, a plot of the data on chi-square probability paper (two degrees of freedom), which represents the exponential distribution, was made. Histograms were prepared in some of these cases.

The computer program prepared for the analysis makes use of appropriate routines from the International Mathematical and Statistical Library (IMSL) for the chi-squared test. The program calculates from the data such parameters as the mean, variance, and percentiles (in this case the 50th, 90th, and 95th) for the exponential and log-normal distributions, which are defined in the program. It also computes the percentage difference for each parameter for comparison purposes, and it is used to compute and print out the approximate frequencies (expected value of the ordered observations) for plotting purposes.

The theory and concepts related to corrective maintenance repair time are given in Section II, in which the relationship between time to repair and the effectiveness of a system is considered.

Statistical considerations, which include a description of the statistical distributions, probability plotting, and testing of distributional assumptions is given in Section III. This describes the theory related to the analysis. Section III also includes a description of the analysis process and the major functions of the computer program used in it. A more detailed description of the

program, its major subroutines, and a definition of the input data formats is given in Appendix F.

The reports from which the data sets were taken are discussed in Section IV. The final two sections include the results of the analysis, a discussion of some of the cases and their results, and the conclusions reached based on the analysis. Recommendations regarding continuing research are presented at the end of Section VI.

II. CORRECTIVE MAINTENANCE REPAIR TIME AND RELATED CONCEPTS

A. CONCEPTS OF SYSTEM EFFECTIVENESS

System effectiveness is a measure of how well a system performs its intended functions and its ability to be retained in or restored to an effective usable condition. In other words, system effectiveness is concerned with the availability of the system to perform its mission successfully in its intended environment [Ref. 6].

Because there are many semantic difficulties in talking of the system effectiveness and the relationship between its components (Figure 1), the following terms are generally recognized components of system effectiveness [Ref. 6]:

(a)- The performance capability of the system.

(b)- The operational readiness or availability of the system, that is, its ability to start performance of a mission when called upon to do so.

(c)- The system dependability, or its mission reliability, that is, its continued capability to perform.

Figure 1 shows that maintainability (downtime) contributes its part to availability, which together with operational readiness are components of system effectiveness. This concept of system effectiveness, one among various concepts which have been developed, was delinated by personnel of ARINC Research Corporation [Ref. 7]. Basically, the

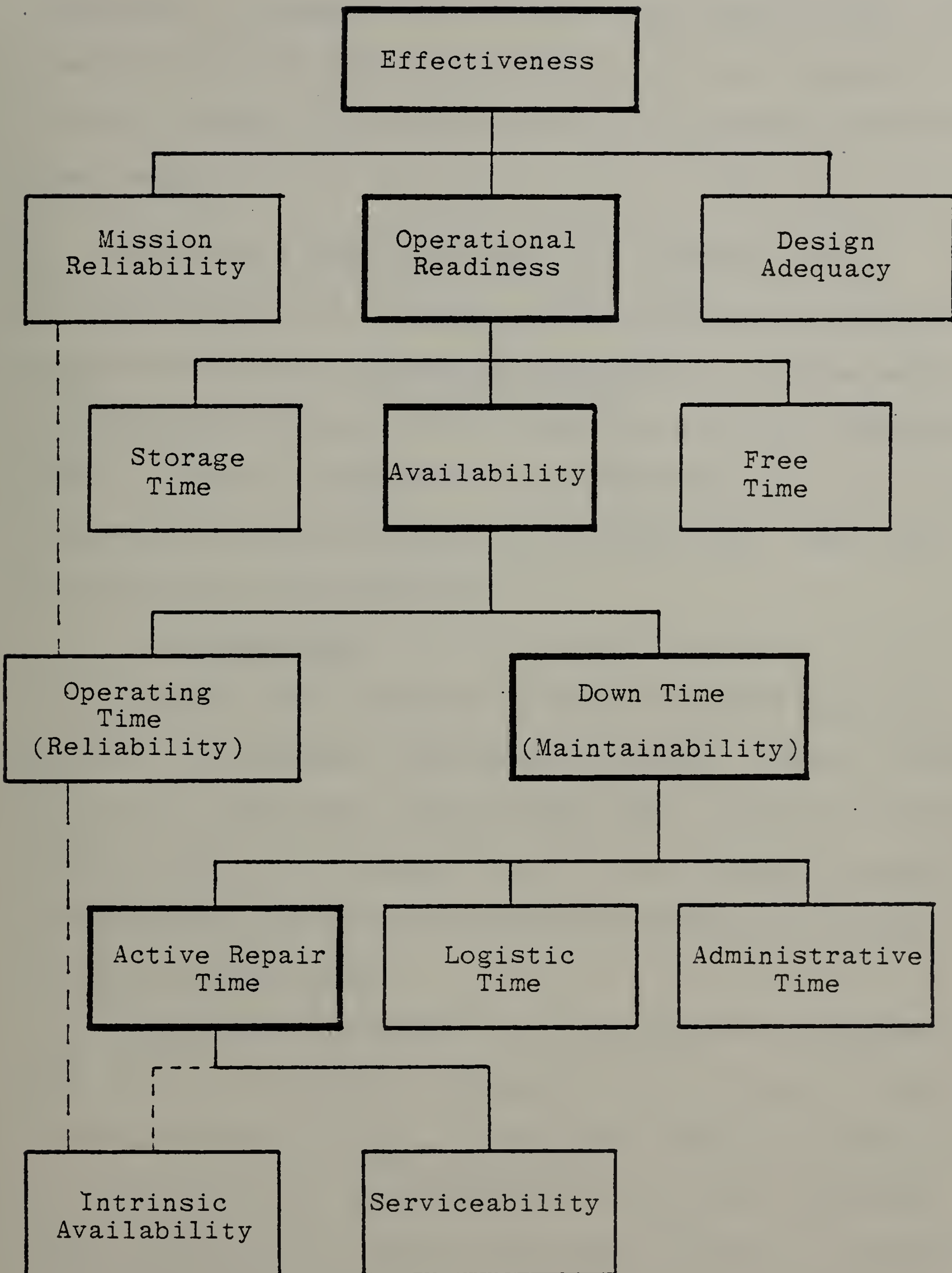
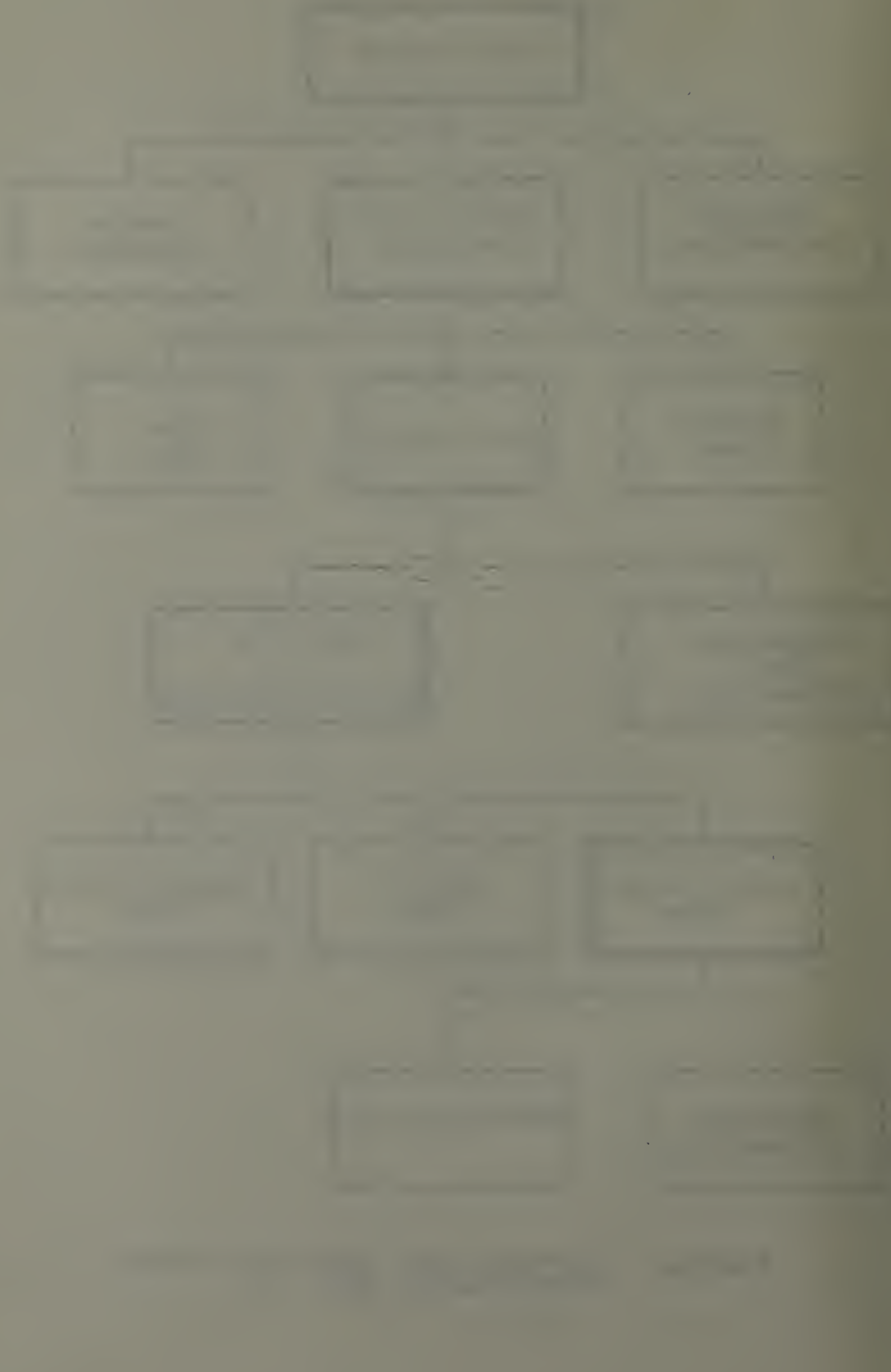


Figure 1: Concepts Associated with Systems Effectiveness [Ref. 7]



components of system effectiveness are probabilities, which, combined together with time measures and environmental conditions, define the system's ability to perform successfully when needed.

B. AVAILABILITY AND MAINTAINABILITY CONSIDERATIONS

Availability is a measure which relates reliability and maintainability to operational readiness. In some cases, availability and operational readiness have been considered to be the same. These are all requirements which must be satisfied during the design of a system, and, thus, have to be quantitatively evaluated.

1. Availability

There are a variety of ways of expressing availability. In general, availability relates "uptime" (reliability) to "downtime" (maintainability), and it may be defined as the ratio between the time the system is capable of performing its mission to the total time the system is in operational demand.

The two expressions of availability of greatest concern are-- (1) inherent availability (A_i), and (2) operational availability (A_o). The latter, which includes in the calculation of the availability ratio all the delay times and the actual active downtime (Figure 1), including preventive maintenance is beyond control of the system designer or producer. Therefore, the inherent availability, which is a hardware oriented measure, is the one which is

usually specified and required within the maintainability contract requirements.

a. Inherent Availability (A_i)

Inherent availability, which includes only intrinsic design variables controllable by the system designer, may be expressed as:

$$A_i = \frac{MTBF}{MTBF + MTTR} \qquad (1)$$

where

MTBF = Mean Time Between Failures

MTTR = Mean Time To Repair

It may be defined as the <u>probability</u> that a system, when used under stated conditions, without consideration of any scheduled or preventive action, and in an <u>ideal</u> support environment, will operate satisfactorily at any given time [Ref. 8].

b. Operational Availability (A_o)

Operational availability, which includes all the delay times as part of the downtime (Figure 1), may be expressed as:

$$A_o = \frac{MTBM}{MTBM + MDT} \qquad (2)$$

where

MTBM = Mean Time Between Maintenance

MDT = Mean Down Time (Including supply and administrative delays and actual active-corrective and preventive maintenance-downtime, during the same time interval)

It may be defined as the probability that a system, when used under stated conditions and in an actual supply environment, will operate satisfactorily at any given time [Ref. 8].

2. Maintainability

Maintainability is a characteristic of system design which determines the ability to keep an operating system in operation (preventive maintenance), or to restore it to a usable condition (corrective maintenance). It is defined in MIL-STD-721B [Ref. 9] as "....the probability that an item will be retained in or restored to a specific condition within a given period of time, when the maintenance is performed in accordance with prescribed procedures and resources."

From its definition, maintainability is concerned with both preventive and corrective maintenance. However, many of the critical problems are related to corrective maintenance, since this involves a "repair" action, often during a mission and within a relatively short period of time. Therefore, time, as a critical factor in corrective maintenance, is an important parameter in maintainability design which should be directed such that the maintenance task times will be minimized. The extent to which the time factor is considered during design depends on the ability to predict, allocate, and demonstrate its quantitative value. In order to do this, statistical methods are used for prediction and evaluation. Some of them are based on the assumption of an underlying distribution of the repair time, and others are "distribution free" methods.

a. MTTR in Corrective Maintenance

The system downtime, from failure occurrence to system restoration to an operating condition, usually includes corrective and preventive maintenance downtimes and delay times. The delay or waiting time includes administrative time and supply time, which to a large extent, are not design controllable.

The Mean Time to Repair (MTTR), a parameter often used for maintainability prediction and maintainability demonstration, is defined in MIL-STD-721B [Ref. 9] as the total corrective maintenance time divided by the total number of corrective maintenance actions during a given period of time. Further, the repair time consists of the actions required to perform on-line repair of a failed item of equipment. These actions, called corrective maintenance tasks, may be separated into four sequential time phases as follows [Ref. 10]:

(a) Detection time - the time to detect or to recognize the existence of a fault.

(b) Diagnostic time - the time to localize and to isolate the fault.

(c) Corrective time - the time to remove and replace the item or to repair it.

(d) Verification time - the time to verify, by testing and alignment, that the fault has been corrected.

The corrective maintenance downtime may also be divided into two steps, the first consisting of the detection time and the second of the <u>active repair time</u>, which includes the

remaining three phases of corrective maintenance as illustrated in Figure 2. Active repair time can usually be described by a statistical distribution and its mean, the MTTR, can be estimated by statistical methods.

b. Maintainability Demonstration

Maintainability Demonstration is a specific test program to be performed, as part of system acceptance testing. Such a demonstration determines the degree to which the specified maintainability requirements have been met.

MIL-STD-471A [Ref. 3] provides methods for demonstrating repair time parameters, such as, MTTR, M'_{max} - allowable maximum maintenance time, $\bar{M}_{pt}$ - mean preventive maintenance time and the median of the repair time distribution. $\bar{M}_{ct}$ - mean corrective maintenance time, referenced in MIL-STD-471A [Ref. 3], is the same parameter as MTTR.

Based on References 8 and 10, the most used test methods in MIL-STD-471A for repair time parameters are methods 4, 8 and 9. These are summarized in Appendix A.

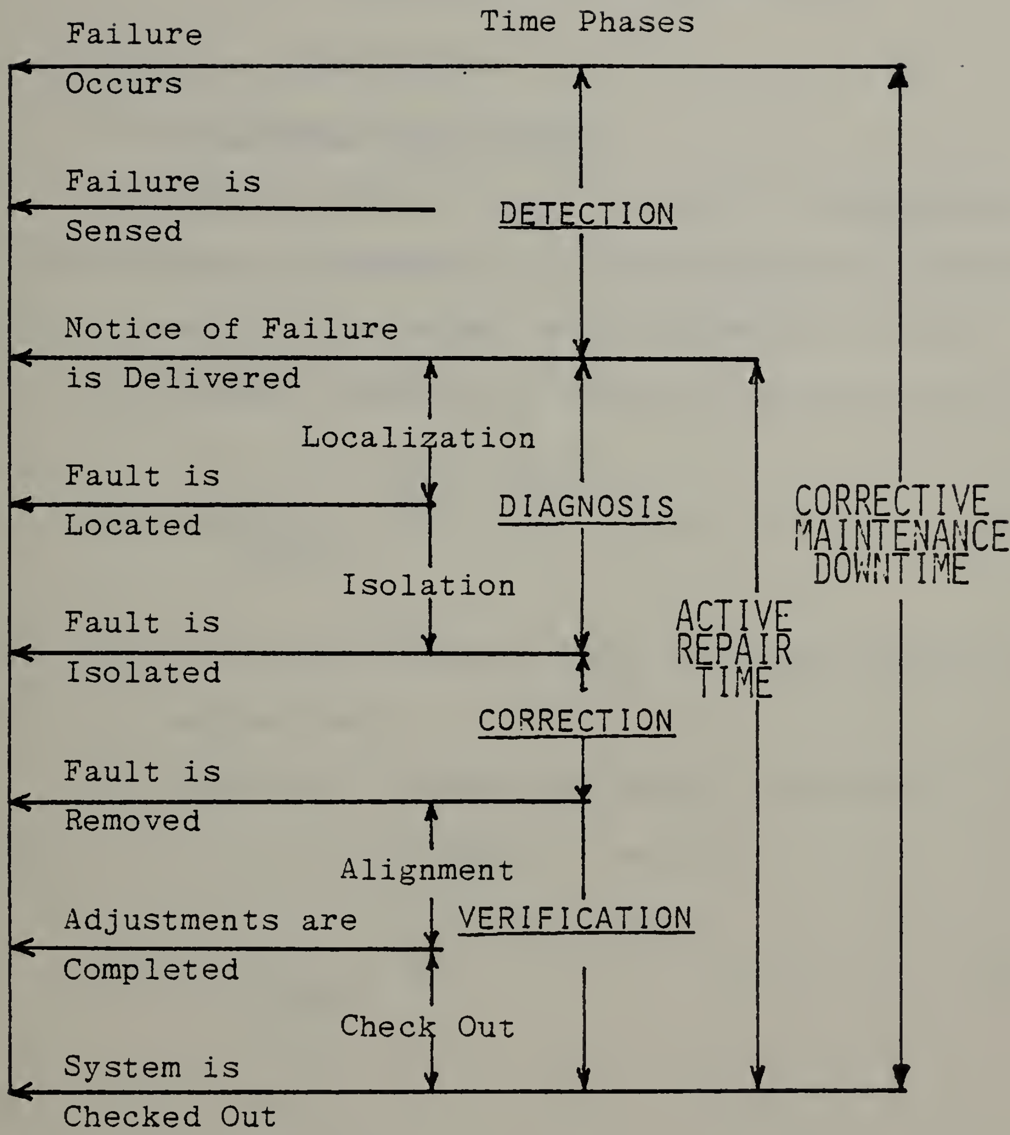


Figure 2: Major Events and Activities Comprising Corrective Maintenance [Ref. 10]

III. STATISTICAL CONSIDERATION

A. STATISTICAL DISTRIBUTIONS

1. The Lognormal Distribution

A random variable is said to have a logarithmic normal distribution (lognormal) if the logarithm of the variable is normally distributed, with parameters μ and σ.

The normal probability density function for y is:

$$g(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(y-\mu)^2} , \quad -\infty < y < \infty \qquad (3)$$

where μ and σ are the expected value and the standard deviation of y, respectively.

The lognormal probability density function for the variable x (y = ln x) is [Refs. 1 and 2]:

$$f(x) = \frac{1}{\sigma x\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\ln x-\mu)^2} \qquad x \geq 0 \qquad (4)$$

where $\mu = \overline{\ln x}$, $\qquad -\infty < \mu < \infty$

and $\sigma^2 = \mathrm{var}(\ln x)$ $\qquad \sigma > 0$

This distribution has many different shapes for non-negative variates. It is skewed to the right, the degree of skewness increasing with increasing values of σ. μ and σ are scale and shape parameters respectively and not location and scale parameters as in the normal distribution.

The lognormal distribution has been shown to be applicable to many economic and biologic processes, when the observed value is a random proportion of the previous value. It is also applicable when the geometric mean better describes the central tendency of the distribution rather than the arithmetic mean [Refs. 1 and 2].

2. Estimation of the Lognormal Distribution Parameters

Parameters of statistical distributions can be derived analytically using standard statistical techniques.

The lognormal distribution, in its simplest form, is a two-parameter distribution, μ and σ. These are estimated as follows [Ref. 4]:

$$\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} (\ln x_i) \qquad (5)$$

$$\hat{\sigma} = \sqrt{\frac{\sum_{i=1}^{n} (\ln x_i)^2}{n} - \frac{\left[\sum_{i=1}^{n} \ln x_i\right]^2}{n^2}} \qquad (6)$$

It can be shown [Refs. 1 and 2] that if x_q and ν_q are the percentiles of order q of $x(\mu,\sigma^2)$ and of $z(0,1)$, the standardized normal distribution, then

$$x_q = e^{\mu+\nu_q\sigma} \qquad (7)$$

From equation (7), the following relations hold:

Mode: $X_M = e^{\mu-\sigma^2}$

Median: $X_{0.5} = e^{\mu}$

Mean: $X_m = e^{\mu+0.5\sigma^2}$

90th Percentile: $X_{0.9} = e^{\mu+1.282\sigma}$

95th Percentile: $X_{0.95} = e^{\mu+1.645\sigma}$

Figure 3 shows the lognormal distribution and its significant parameters.

The parameters of the lognormal distribution can also be derived from a straight line obtained from and fitted to data points plotted on lognormal graph paper [Ref. 11].

The estimate of the parameter $\mu, \hat{\mu}$, is found first by entering the plot at the 50th percent point on the probability scale of the paper, and by reading the value of the variable on the other scale. The natural logarithm of this value, which is the estimate of the median of the distribution function, is the estimate of μ.

The estimate of σ is found in two steps. First the value of the natural logarithm of the 84th percentile, $x_{0.84}$, is found, then the estimate of σ is the difference between $\ln x_{0.84}$ and $\hat{\mu}$.

3. The Exponential Distribution

The probability density function of the exponential distribution is [Ref. 4]

$$f(x,\lambda) = \lambda e^{-\lambda x}, \qquad x \geq 0,\ \lambda > 0 \qquad (8)$$

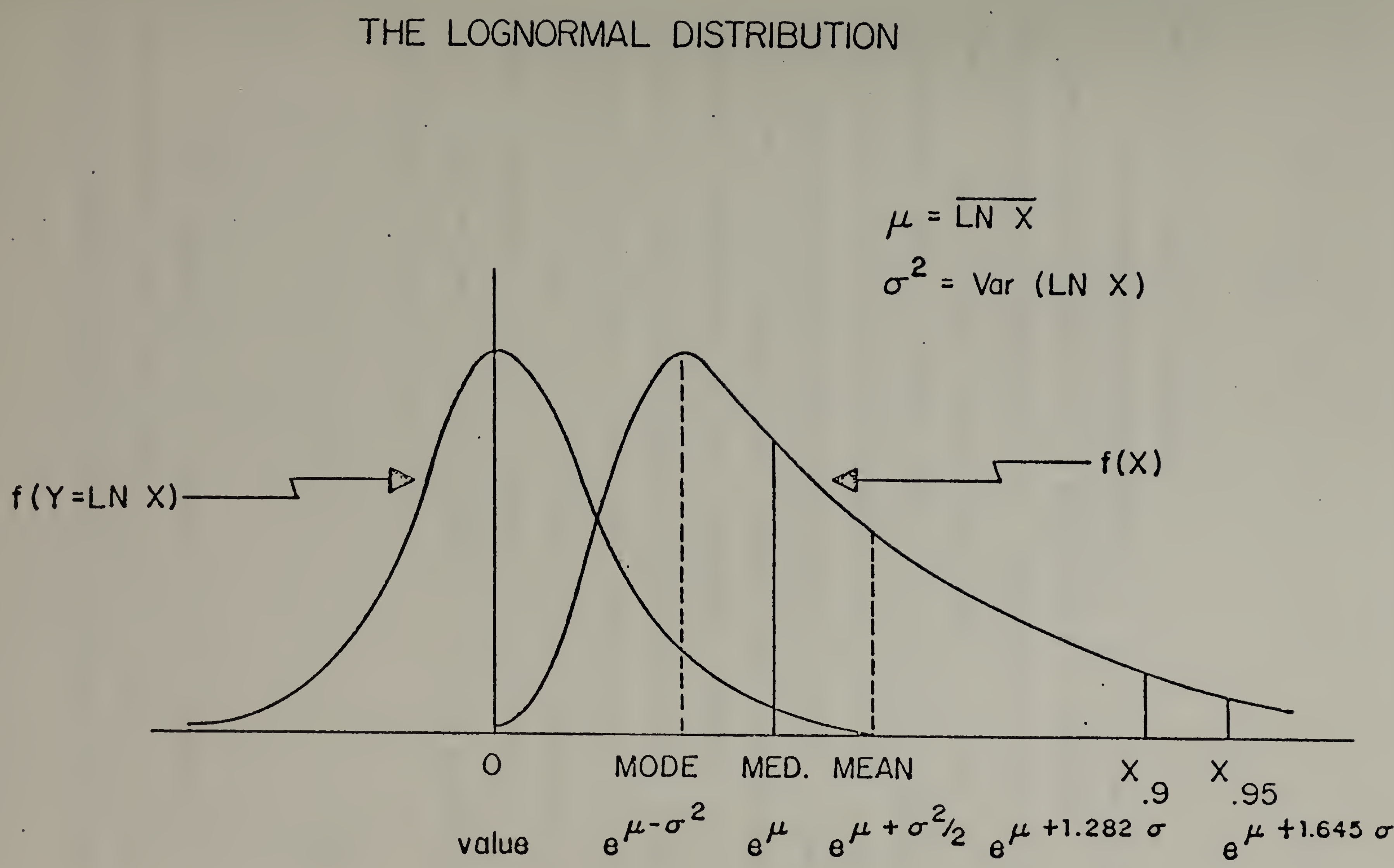


Figure 3: The Lognormal Distribution [Ref. 2]

Its cumulative distribution function is

$$F(x,\lambda) = \int_0^x \lambda e^{-\lambda t} \, dt = 1 - e^{-\lambda x} , \qquad (9)$$

which can be easily evaluated.

The exponential distribution is used frequently as a time-to-failure model for a system when a constant failure rate is assumed [Ref. 4]. But for the case of repair times, it can be shown heuristically that it is not an appropriate model. Because repair time includes diagnostic, correction, and verification tasks, involving time, a repair time distribution must have a value of zero at time t=0, increases to its maximum value rapidly and then gradually decreases towards zero as time increases. However, an exponential distribution suggests that the maximum number of repairs can be made in zero time. When repair times are clustered or grouped into intervals (e.g. histograms), the result may appear to fit an exponential distribution.

The estimator of the parameter λ of the exponential distribution from given data is

$$\hat{\lambda} = \frac{1}{\bar{x}} = \frac{n}{\sum_{i=1}^{n} x_i} \qquad (10)$$

The parameter of the exponential distribution can also be derived from data plotting on chi-square (two degrees of freedom) probability paper, by reading the value

of the variable at the 50th percent point, $x_{0.5}$. The exponential cumulative distribution function (9), for $x_{0.5}$ is

$$F(x_{0.5}, \lambda) = 1 - e^{-\lambda x_{0.5}}$$

thus

$$\hat{\lambda} = - \frac{\ln 0.5}{x_{0.5}} = \frac{0.693}{x_{0.5}}$$

The estimation of other percentiles, like the 90th and the 95th which are often used for allowable maximum repair time, can also be derived from (10) and the estimator of λ.

For example, the 90th percentile is

$$\hat{x}_{0.9} = - \frac{\ln 0.1}{\hat{\lambda}} = \frac{2.3}{\hat{\lambda}}$$

or

$$\hat{x}_{0.9} = - \frac{\ln 0.1}{\ln 0.5} x_{0.5} = 3.32 \; x_{0.5}$$

B. PROBABILITY PLOTTING AND TESTING OF DISTRIBUTIONAL ASSUMPTIONS

1. Probability Plotting

Plotting data points on probability paper is quite simple and does not require complicated calculations or the use of statistical tables. According to Hahn and Shapiro [Ref. 4], "Probability plotting is a subjective method in that the determination of whether or not the data contradict the assumed model is based on a visual examination, rather

than a statistical calculation." The only calculation needed is that of the expected values of the ordered observations, which approximate the cumulative distribution function. (A more detailed discussion of the expected value of an ordered observation is given in Appendix B.)

As mentioned in the previous section, a plot of the data, when it fits the assumed probability distribution, can provide estimates of the percentiles of the distribution and its parameters.

When plotting the data on special graph paper designed for the assumed distribution, and if the assumed distribution is correct, the plotted points will tend to fall in a straight line except for extreme value points, discussed in Section VC. If the assumption is inadequate, the plot will not be linear; the variations of the data points from a straight line will be significant [Refs. 4, 11 and 12].

The selection of the appropriate distribution should be based on an understanding of the underlying physical phenomena.

In this study, the lognormal distribution is assumed to be the underlying distribution for corrective maintenance repair times and therefore logarithmic normal probability paper was used. The assumption of the exponential distribution and the use of chi-square (two degrees of freedom) probability paper, were only made where the statistical tests indicated close results for both distribution assumptions.

or where other considerations indicated that the exponential distribution appeared to be appropriate.

If the plot deviates significantly from a straight line, the assumed distribution does not adequately describe the data. Systematic deviations are indication that the model is inadequate. The determination of what can or cannot be considered a straight line is a subjective matter. The larger the sample size and the greater the divergence from the assumed distribution, the easier it is to detect non-random deviations.

In most statistical texts, the method of least squares is suggested for fitting a straight line to plotted data. However, fitting "by eye" may be sufficient because in the end a subjective decision on whether or not the assumed model is adequate must still be made. Furthermore, the method of least squares is not appropriate in the case of probability plotting, because the ordered observations are not independent. The procedure for preparing a probability plot from a given set of data involves ranking of the observations in ascending order and plotting the i-th ordered value versus $\frac{(i-\frac{1}{2})}{n}$ x 100, which is the expected value of the i-th observation. Thus, there is a constraint that $x_{i+1} \geq x_i$ for all i, and therefore the ordered observations are not independent (see Appendix B).

Figures 4 and 5 readily illustrate the capability of probability plots to give a quick indication of the suitability of a distribution.

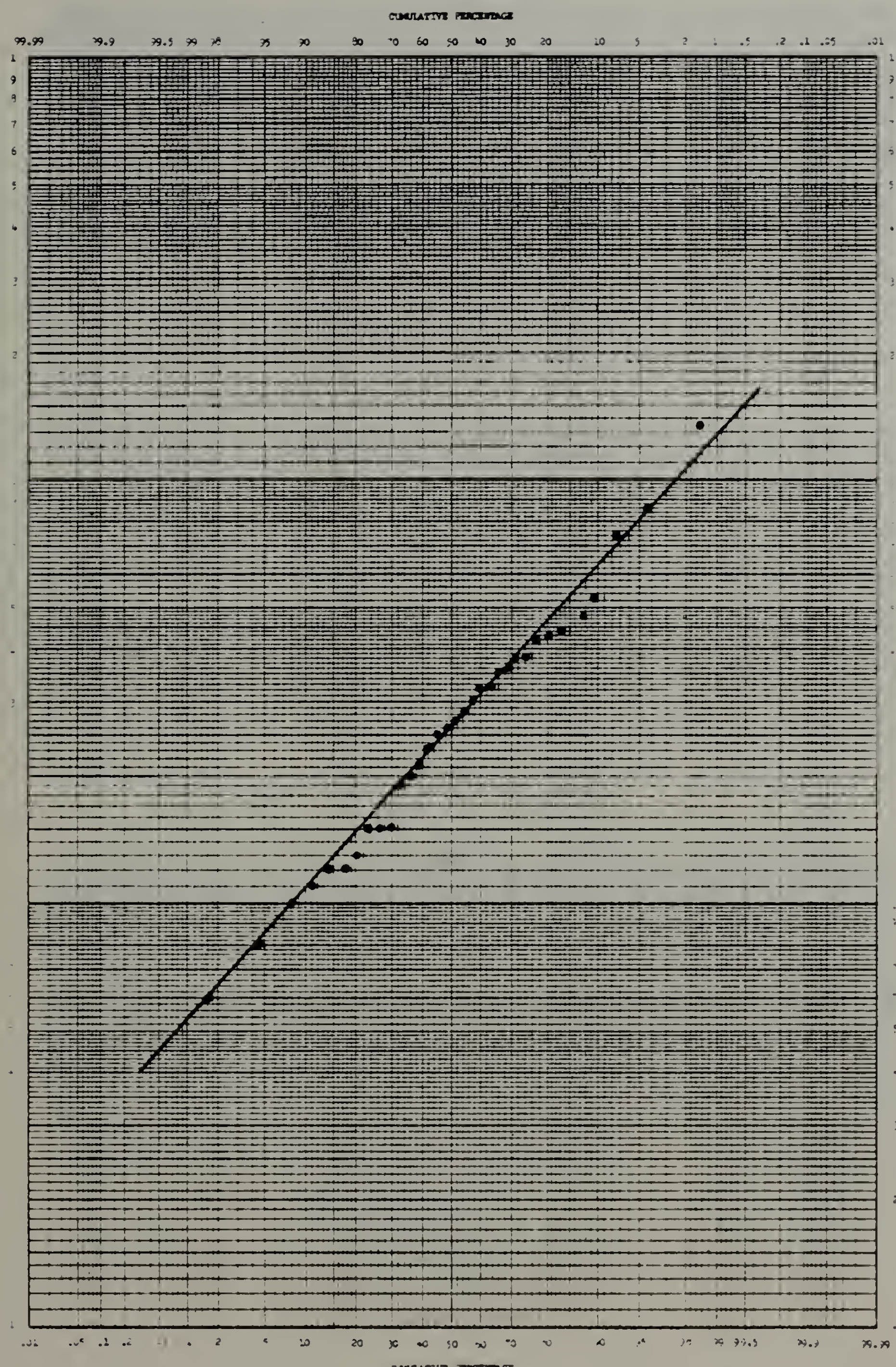


Figure 4: Lognormal Probability Plot of Lognormally Distributed Data [Set No. 19]

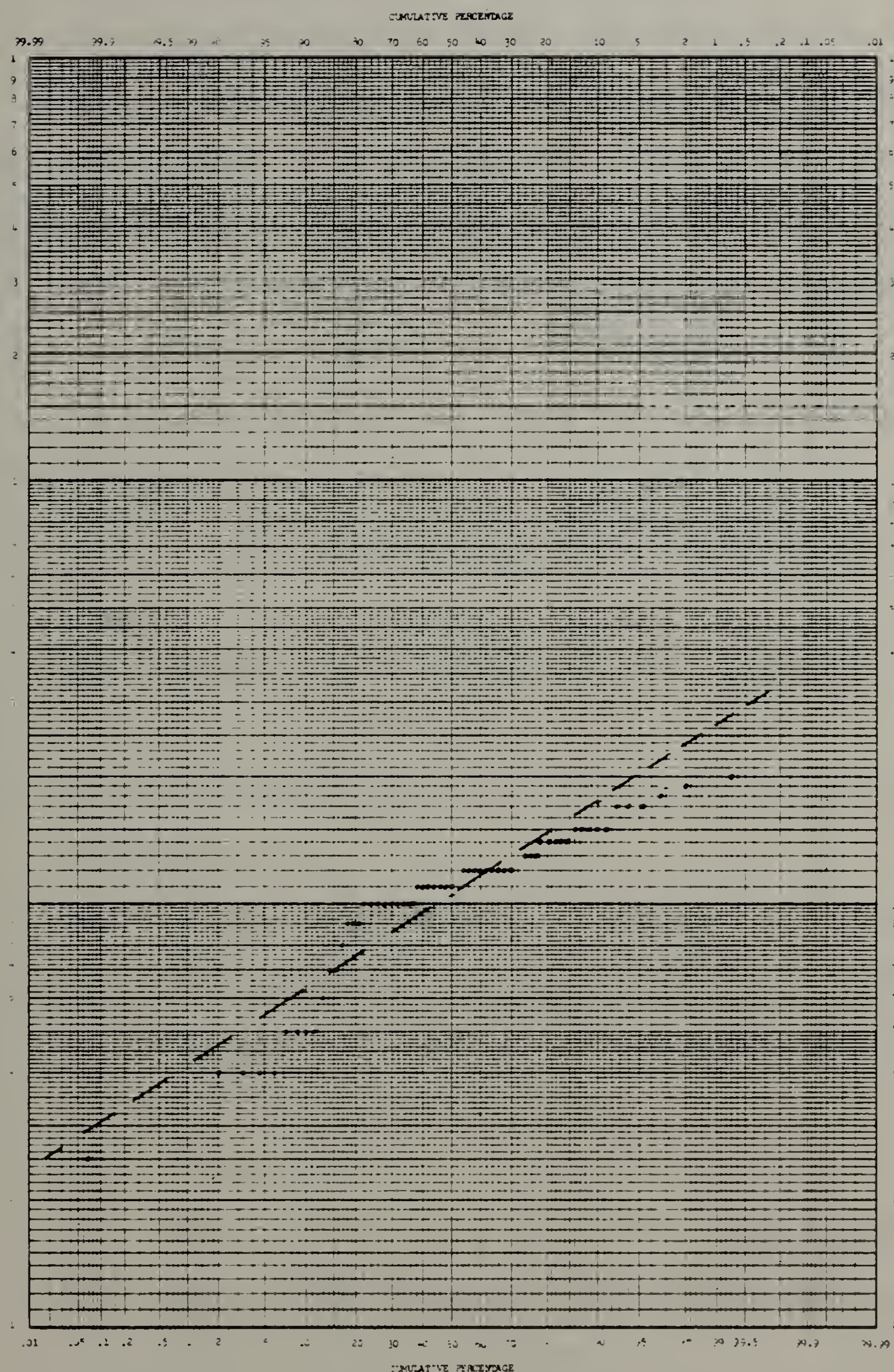


Figure 5: Lognormal Probability Plot of Non-Lognormally Distributed Data [Set No. 5]

2. Tests for Distributional Assumptions

The second technique which has been used in order to analyze the assumption about the lognormal distribution is statistical test of significance. Such a test of a distributional assumption provides an objective technique, to some extent, for assessing whether or not an assumed model provides an adequate description of observed data.

There are three basic steps involved in a statistical test [Ref. 4]:

(1) A test statistic is calculated from the observed data.

(2) The probability of obtaining the calculated test statistic is determined.

(3) Assessment is made of the adequacy of the assumed distribution.

   (a) If the probability of obtaining the calculated test statistic is "low", one can conclude that the assumed distribution does <u>not</u> provide an adequate representation.

   (b) If the probability of obtaining the calculated test statistic is not "low", <u>then the data provide no evidence that the assumed distribution is not adequate</u>.

The definition of "low" or "not low" depends on the user's preferences and the consequences of rejecting the distribution. Since a probability of 0.1 or 0.05 or less is usually said to be low, the probability of 0.05 was selected as the reject criterion.

The above steps differ slightly from the more usual procedure in which the test statistic is compared to a value which is such that the area under the distribution to its right is equal to the selected level of significance (i.e., the reject criterion is the value of the variable such that the probability of not exceeding this value is equal to one minus the level of significance). If the test statistic is greater than this value, the assumption can be rejected at the given level of significance [Ref. 13].

It should be pointed out that a statistical test, although it allows one to reject an assumption as inadequate, does not allow one to prove that the assumption or the distribution is correct.

In this study, the parameters of the distribution for corrective maintenance repair times are not known and had to be estimated from the data. The two statistical tests suggested by Hahn and Shapiro [Ref. 4] were used. The first one is the conventional chi-squared goodness-of-fit test [Ref. 14], and the second one is a test developed by Shapiro and Wilk [Ref. 5], called the W-test.

The use of the W-test to evaluate the assumption of a lognormal distribution was done (manually) for those sets of data in which the number of data points was not more than 50 (due to unavailability of tables for samples of size greater than 50).

3. <u>The Chi-Squared Goodness-of-Fit Test</u>

This test is one of the oldest and most commonly used for evaluating distributional assumptions. Basically,

the given data are grouped into frequency cells and compared to the expected number of observations based on the assumed distribution. The test statistic, calculated from this comparison, will tend to exceed a chi-square variate if the assumed distribution is not correct.

The advantage of this test is that it can be applied to test any distributional assumption without having to know the values of the distribution parameters. These have to be estimated as part of the test procedure. Its disadvantages are its lack of sensitivity in detecting inadequate assumptions when the number of observations is small, and the need to arrange the data into arbitrary number of cells ("equiprobable cells"), which determines the number of degrees of freedom and can affect the result of the test.

There are two methods for dividing the data into classes or cells: one is applicable when the data are originally arranged in frequency classes and thus, there is no need to determine the number of cells since the original number of frequency classes is used. The other method, used in this study and described below, applied when the data are not initially tabulated in classes. In this case, the number of cells is arbitrary. Since the number of observations in the samples used is small (less than 200), the rule suggested by Hahn and Shapiro [Ref. 4], to use a number of cells as large as possible, subject to the restriction that it must not exceed n/5, (n - the sample size), the number of cells used in this study is an integer less than or equal to n/5.

The computations involved in the chi-squared goodness-of-fit test were made by a computer program, in which were given, as inputs, the assumed cumulative distribution function, the number of observations, the number of equiprobable cells and the number of parameters estimated from the sample. The outputs were the chi-square statistic and its probability of exceeding a chi-square variate for a given number of degrees of freedom.

In order to present the basic calculations involved in the test, the procedure used is described as follows [Ref. 4]:

(a) The cells boundaries are determined from the assumed cumulative distribution as the values such that the probability of the observation value falling within a given class is 1/k for each class:

$$p_r[x \leq x_i] = \frac{i}{k} \tag{11}$$

where x - the random observation to be assigned to the i-th cell

x_i - the i-th cell boundary to be solved from (11)

k - the number of cells, in this case equal to n/5

The lower bound of the first cell and the upper bound of the last cell are the smallest and largest values that the observations (the repair time) may take on.

(b) The expected number of observations for each cell E_i is equal to n/k (= five in this case) for each cell.

(c) The number of observed values in each cell, M_i, is counted based on the results of equation (11).

(d) The test statistic is

$$X^2 = \frac{k}{n}\left[\sum_{i=1}^{k} M_i^{\,2}\right] - n \qquad (12)$$

(e) The computed value X^2 is used to compute the level of significance, or the probability of a chi-square variate with ν degrees of freedom (equal to k-m-1, where m is the number of parameters estimated from the sample) to exceed the calculated X^2:

$$\alpha = P_r[\chi^2_{1-\alpha}(\nu) \geq X^2] \qquad (13)$$

If α is less than or equal to 0.05 the assumed distribution can be rejected as inadequate.

4. <u>The W Test to Evaluate the Assumption of a Lognormal Distribution</u>

The W test is shown in Reference 5 to be an effective procedure for evaluating the assumption of normality against non-normal alternatives, even if only a relatively small number of observations are available. Hahn and Shapiro [Ref. 4] suggest that the W test may also be used to evaluate the assumptions of a lognormal distribution. This follows because if the logarithms of the observations follow a normal distribution, then the original values of the observations are lognormally distributed.

The following is the procedure applied while using tables from Reference 4 and which are reproduced and given in Appendix C.

(a) The observations are ordered such that $x_1 \leq x_2 \leq \ldots. \leq x_n$.

(b) The following parameters (for the lognormal assumption) are computed

(i)

$$S^2 = \sum_{i=1}^{n} (\ln x_i)^2 - \frac{\left[\sum_{i=1}^{20} \ln x_i\right]^2}{n} \qquad (14)$$

(ii) If n is even, k = n/2; if n is odd, k = (n-1)/2. Then,

$$b = \sum_{i=1}^{k} [a_{n-i+1}(\ln x_{n-i+1} - \ln x_i)] \qquad (15)$$

where the values of a_{n-i+1} for i=1,...,k are given in Table VII (Appendix C), for $3 \leq n \leq 50$.

(iii) The test statistic, W, is

$$W = \frac{b^2}{S^2} \qquad (16)$$

(c) The approximate probability of obtaining the calculated value of W can be obtained from Table VIII (Appendix C) or from:

$$z = \lambda + \eta \ln[\frac{W-\varepsilon}{1-W}] \qquad (17)$$

using the values of λ, η and ε given in Table IX (Appendix C) for the appropriate sample size, and then using the standardized normal distribution to determine the probability of obtaining a value less than or equal to z, which is the significance level of the test:

$$\alpha = P\ [Z_{\alpha} \leq z] \tag{18}$$

If α is less than or equal to 0.05, the selected level of significance in this study, the lognormal distribution can be rejected as an inadequate assumption.

Hahn and Shapiro [Ref. 4] also suggest the use of a test for the assumption of an exponential distribution, called the WE test. In this test, used in some cases in this study, the WE statistic calculated from the data is compared against a 90% or 95% range (equal to a significance level of 0.1 or 0.05 respectively), which is defined by a lower and an upper point. If the WE statistic falls outside this range, i.e. a too-high or too-low value, it indicates non-exponentiality. A detailed description of the test procedure is given in Reference 4.

C. THE ANALYSIS PROCESS

The methods and techniques described were used primarily in order to analyze each set of data for the assumption for the lognormal distribution. The exponential distribution

assumptions was also tested.

Basically, the process for the analysis for each set of data has been the following:

(1) Review of the data and preparation for computer run (keypunching).

(2) Computer run, which includes the following functions:

(a) Sorting the data in ascending order.

(b) Computation of the "approximate cumulative frequency" (or the expected value of the ordered observations), using $\frac{i-0.5}{n} \times 100$ for the i-th observation, where n is the sample size (see Appendix B).

(c) Printout of a table of the data (repair times) and their plotting position points in ascending order, for plotting purposes.

(d) Calculation and plotting of the theoretical and sample CDF, based on the exponential and lognormal distributions. (This is done by a routine from the IMSL package.)

(e) Chi-squared goodness-of-fit test for the assumptions for the exponential and lognormal distributions, using a routine from the IMSL package which gives the value of the test statistic and the probability of obtaining it for a given distribution.

(f) Calculation of the sample mean and variance, the lognormal distribution parameters and the median,

90th and 95th percentiles based on both the exponential and the lognormal distributions parameters.

(g) Calculation of the percentage error between the results of the mean and percentiles of the exponential distribution relative to those of the lognormal distribution.

(h) Printout of the above results in a summary table.

(3) Based on the plotting positions (item c above), each set was plotted on lognormal probability paper. A line, which represents the "best" fit to the data points was drawn.

(4) Estimated parameters and percentiles from the plot were determined. The percentage errors between the results from the plot and those from the theoretical lognormal calculation were calculated.

(5) A W-test was performed for the lognormal distribution assumption, for those sets which have up to 50 data points.

(6) An analysis of the results, based on both the statistical tests and the probability plot was performed.

(7) As a result of the analysis, the need for further statistical tests, such as testing for normality or a WE-test for exponentiality [Ref. 4], was determined. Also when needed, a probability plot on

chi-square (two degrees of freedom) probability paper was made and in some cases a histogram was drawn, in order to get insight to the shape of the sample distribution.

IV. DATA SOURCES DESCRIPTION

A. GENERAL

All of the repair times used in the analysis come from maintainability demonstration reports for electronic systems/equipment. In most of the cases, these reports include a description of the tests used during the maintainability demonstration. These details have been examined in order to analyze specific repair times that vary, to some extent, from the expected value.

All the data sources include the various elements of repair time such as diagnostic (localization and isolation), removal/replacement, verification and check-out time. The time elements are combined in the reports differently, depending on the nature of the demonstration test and the equipment.

Although one of the objectives of this study was to test the lognormal distribution for corrective maintenance repair time of mechanical systems/equipments, repair time data for such systems were not obtainable.

The systems/equipments analyzed are listed in Table I, which includes the source references.

The following is a discussion of the data source reports.

TABLE I

Systems/Equipments Analyzed

No.	Description	Source Reference
1	AN/TRC-87 Communications Transceiver	15
2	Quick Reaction Capability Radar	15
3	AN/GSA-51 Back Up Interceptor Control System	15
4	AN/FPS-80 Tracking Radar	15
5	AN/TPS-39(V) Radar Surveillance System	15
6	AM/3949-GR Radio Frequency Amplifier	15
7	AN/ARC-164(V) Radio Set	16
8	AN/ASN-131 Airborne Navigation System-- Inertial Measurement Unit (IMU) Interface Electronics Unit (IEU)	17
9	HARPOON Ship Command Launch Control Set	18
10	Defense Communication System Satellite Control Facility Interface System (DSIS)	19
11	USASATCOMA Communication Subsystem (Contingency Configuration)	20
12	USASATCOMA Communication Subsystem (Nodal Configuration)	21
13	Continental Air Defense Command Ground Data System (User Display Segment)	22
14	Strategic Air Command Ground Data System (User Display Segment)	23
15	SAMSO 46 FOOT TT&C Antenna	24
16	NADC Digital Television Projection Unit	25
17	USASATCOMA HT/MT Terminal	26
18	National Military Command System Ground Data System (User Display Segment)	27
19	US Army Electronics Command AUTODIN Memory/Memory Control Equipment	28

B. DISCUSSION OF SOURCE REPORTS

(1) RADC Case Histories in R & M Demonstrations [Ref. 15]

This paper provides tables of repair times for six electronic systems. The purpose of the paper was to discuss case histories of reliability demonstration and of maintainability demonstration. The maintainability demonstrations generally supported the assumption of the lognormal distribution of repair times. However, deviations were observed in some cases, in both the paper and in this study.

The only method used in the paper for the purpose of "statistical analysis", which was not the purpose of the paper, was the use of histograms of the number of repair actions versus the time required to finish a repair action. A histogram alone, even though it may provide some ideas about the overall shape of the data distribution, is not an accurate technique to assess whether a particular distribution fits the data or not.

The criterion for success or failure of maintainability demonstration, in most of the cases, depend on whether the demonstrated mean time is or is not less than the required mean time to repair. This criterion, satisfactory for some purposes, is not what was used in this study to determine whether a set of data follows the pattern of a lognormal distribution or not. Fortunately the authors of the paper provided tables of maintenance actions times for each case, ".....for anyone who wishes to perform a more detailed statistical analysis".

(2) R/M Assessment and Demonstration Test Report on AN/ARC-164(V) Radio Set [Ref. 16]

This report includes, in detail, the maintainability demonstration test data sheets for organizational and intermediate levels. The system consists of four Line-Replaceable Units (LRU's). The number of maintenance tasks was in accordance with Appendix A of MIL-STD-471 (Replaced by MIL-STD-471A-Ref. 3).

The purpose of the maintainability demonstration test, as stated in the report, was "to demonstrate compliance to the quantitative maintainability requirements specified (for the system)". Method 2 of MIL-STD-471 (which is now method 9 of MIL-STD-471A Ref. 3), was used to determine the accept/reject criteria.

In this study only the 50 repair actions for intermediate level were tested and analyzed.

(3) AN/ASN-131(SPN/GEANS) - Maintainability Assessment and Demonstration on Final Report [Ref. 17]

The objective of the maintainability demonstration was to evaluate the maintainability characteristics of the AN/ASN-131 Inertial Measurement Unit (IMU) and Interface Electronics Unit (IEU).

Twenty-two organizational level maintenance functions and 22 intermediate level functions were demonstrated. The tests were intended to demonstrate the effectiveness of Built-In-Test-Equipment (BITE) for fault isolation to the LRU level for organizational maintenance and for the printed circuit board or module level for intermediate level

maintenance, which consisted of isolated and repairing faults on printed circuit boards or modules in all major assemblies.

Worksheets containing all raw data were included in the report. This enabled the analysis of extreme points in this study. In order to utilize the data, four separate sets were prepared, one for each of the maintenance levels for IMU and for IEU. Since each of these sets contained a relatively small number of elements, the organizational level and the intermediate level repair times for both of the units (IMU and IEU), were combined (i.e., the analysis was performed on each of the maintenance levels rather than on the subsystems). It is meaningless to use all the 42 times available (in one task on the IMU, the repair times were not available) together because of basic differences in repair actions and times between organizational level and intermediate level.

The report does not include any assessment of the distribution of repair time. The demonstration was performed in accordance with MIL-STD-471, Test Method 3 (which is equivalent to test method 4 in MIL-STD-471A - Ref. 3).

(4) Final Report - Maintainability Demonstration for Harpoon SCLCS [Ref 18]

The maintainability demonstration report for the Harpoon Ship Command-Launch Control Set (HSCLCS) includes a statistical analysis of corrective maintenance repair times and detailed technical discussion which made it possible to filter the data in order to remove anomalies which

biased the data. For example, all the switchboard times or tasks in which the repair time was estimated were eliminated. This is because the switchboard was not part of the equipment demonstrated, but it failed during the demonstration and its repair times were included in the report.

In the data analysis part of the report, there are indications of the suitability of the lognormal distribution to corrective maintenance repair times. This is assessed from histograms and a chi-squared test, which is not presented in the report. These assessments were made in order to determine the maximum expected repair time. It is also pointed out that the "remove/replace" time is the outstanding element.

The statistical analysis was performed in accordance with MIL-STD-471, Test Method 2 (MIL-STD-471A, Test Method 9).

It is said in the report that "The statistical tests on the mean and maximum repair times indicate that the system meet the specified requirements even when the switchboard times are included" and ...."Histograms of the time data and their logarithms show clearly the superiority of the fit of the "lognormal" form over the "normal" form. No sophisticated statistical tests are really necessary in deriving the recommendation/decision to use the "lognormal" form."

Despite the above statements, the analysis performed in this study shows different results, concerning the fit

of the lognormal distribution to the repair times (see Table III in Section V).

(5) <u>DSIS-SCF Maintainability Demonstration Report</u> [Ref. 19]

This report presents a summary of the maintainability demonstration of the Defense Communication System/Satellite Control Facility Interface System (DSIS) for the Satellite Control Facility (SCF). The demonstration consisted of 50 test faults, (25 for on-line repair and 25 for off-line repair). A delay time of two minutes to simulate getting the spare part was charged against the <u>overall restore time</u> for each LRU that was removed and replaced as part of the fault isolation procedure.

The specific Failure Data Sheets used for each test are included in the report. This allowed verification of some data, which were originally rounded-off.

The analytical techniques presented in MIL-STD-471, Test Method 2 (MIL-STD-471A, Test Method 9), were used to determine MTTR.

No assessment or consideration has been made in the report on the distribution of the repair times.

The on-line repair times and the off-line repair times were used separately for the statistical analysis in this study, since it is meaningless to combine them.

(6) <u>Philco-Ford Corp., Western Development Laboratories/Ford Aerospace and Communications Corp. ESD - Maintainability Demonstration Reports</u> [Refs. 20-28]

A number of partial reports of maintainability demonstration, primarily on communication systems and

subsystems, have been used as a source for maintenance action time data. The demonstrations reported in these reports were conducted between 1972 and 1978.

There are some characteristics which appear in almost all these reports:

(a) The maintenance actions were to the LRU level.

(b) The number of tests, 50 in most of the cases, is based on MIL-STD-471, Test Method 2.

(c) The data analysis tends to show that the distribution of corrective maintenance repair time is essentially lognormal. This has been shown by using a plot on lognormal probability paper. (This alone is not sufficient, as is shown in this study.) A straight line through the plotted data was drawn by using the calculated values of the 50th and 90th percentiles, which does not take into consideration expected variations at the extreme points.

(d) In three cases there is an analysis of "achieved" versus "inherent" maintenance times. The reasons for the difference in the maintenance times are related to supply and to availability of proper Aerospace Ground Equipment (AGE). In some cases the difference was due to lack of familiarity with the equipment and the technical documentation.

In general, the conclusions on the underlying distribution of corrective maintenance repair times, were the

same as those reached in this study, except for a few cases, discussed later, in which there are opposite conclusions, with regard to the exponential and lognormal distributions.

Basically, an error was made when plotting the data on exponential graph paper, from which is was concluded that the exponential distribution is a good fit. In one such case [Ref. 28] an "explanation" is given: "....Figure 1 (a semi-logarithmic graph paper) shows that the data is exponentially distributed, which frequently occurs when repair techniques include diagnostics which have clustered running time and component replacement times which are constant." The results of this study do not support this statement as discussed in Section VA2.

In most of the cases, the deviations of the data points from a straight line on lognormal probability paper included in the reports are not significant. However, in two cases [Refs. 20 and 26], the deviations cannot be considered random. The explanation given in Reference 26 may explain many of the deviations in this and other cases:- ".....the constant low maintenance time distribution at the lower end of the graph is caused by the low time fault isolation and "patching" of redundant up and down converters."

V. DATA ANALYSIS RESULTS

A. RESULTS OF TESTS FOR DISTRIBUTIONAL ASSUMPTIONS

1. Summary of Results

Table II summarizes the results of the statistical test analysis. It shows for each case the probability of getting the calculated test statistic, which is the level of significance of the test to be compared to the five percent level chosen as the reject criterion.

In those cases in which the results of the chi-squared test and the W test contradict each other, a plot on lognormal probability graph paper was used to determine the appropriateness of this assumption ((+) in Table II).

Most of the sets of data show that the lognormal distribution cannot be rejected as an adequate descriptor for corrective maintenance repair time. From Table II, the assumption of the exponential distribution is rejected in 17 sets, while in four more sets (sets 3, 14a, 14b and 18a) the probability of getting the chi-square test statistic is less than 0.1. The assumption of the lognormal distribution cannot be rejected in 16 sets, while in four sets (sets 10a, 13b, 14a and 18a), discussed later, the results of the two tests indicate opposite conclusions with regard to rejection of the lognormal assumption. In three sets (sets 5, 9 and 11) both assumptions are rejected and

TABLE II

Probability of Getting Test Statistics

Set No.	Sample Size	Exponential		Lognormal		
		$P[\chi^2]$	Rej.(*)	$P[\chi^2]$	P[W]	Rej.(*)
1	59	0.04	+	0.23	-	
2	20	0.006	+	0.27	0.06	
3	90	0.06		0.94	-	
4	45	0.15		0.92	0.71	
5	75	10^{-24}	+	10^{-9}	-	+
6	38	10^{-5}	+	0.32	0.07	
7	50	10^{-5}	+	0.13	0.06	
8a	21	0.005	+	0.094	0.25	
8b	21	0.04	+	0.71	0.87	
9	44	10^{-3}	+	0.053	< 0.01	+
10a	25	0.006	+	0.11	0.05	(+)
10b	25	0.11		0.11	0.42	
11	50	10^{-7}	+	0.004	< 0.01	+
12	50	10^{-3}	+	0.41	0.06	
13a	50	0.002	+	0.15	0.58	
13b		10^{-8}	+	0.016	0.07	(+)
14a	37	0.096		0.45	0.03	(+)
14b		0.063		0.35	0.26	
15	50	0.014	+	0.17	0.85	
16	22	10^{-4}	+	0.34	0.43	
17	50	0.395		0.006	< 0.01	+
18a	39	0.086		0.047	0.85	(+)
18b		0.05	+	0.15	0.86	
19	33	0.03	+	0.36	0.98	

(*) - Criterion for reject if $P[\chi^2]$ or $P[W] \leq 0.05$.

(+) - To be determined from probability plot/histogram.

and in four sets (sets 3, 4, 10b and 14b) both assumptions cannot be rejected; however the resulting probabilities for the lognormal distribution are much higher. Only in one set (set 17) is the lognormal distribution rejected and the exponential distribution cannot be rejected. The sets in which the lognormal distribution is rejected are sets 5, 9, 11 and 17.

The detailed results of the data analysis and the probability plots for each set are presented in Appendix D.

The following is an example of the computer program summary table which includes the results for the χ^2 goodness-of-fit test and calculated parameters from the sample data.

AN/GSA-51 BACK UP INTERCEPTOR CONTROL SYSTEM

SAMPLE SIZE N = 90		NO. OF CELLS K = 18		(a)
SAMPLE MEAN = 20.43		STANDARD DEV = 17.07		(b)
	EXPONENTIAL	LOGNORMAL	ERROR	
PARAM1	0.05	2.73		(c)
PARAM2		0.57		
MTTR	20.43	20.42	0.06 %	
50-TH PERCNT	14.16	15.33	7.64 %	(d)
90-TH PERCNT	47.04	40.46	16.28 %	
95-TH PERCNT	61.21	53.25	14.94 %	
CHI-SQR STAT	25.60	7.60		(e)
DEG OF FREED	16	15		(f)
SIGNIF LEVEL	0.599E-01	0.939E 00		(g)

Notes:

(a)- The number of equiprobable cells, K, was chosen as $\frac{N}{5}$ where N is the sample size.

(b)- The sample mean and the sample standard deviation are calculated based on the maximum likelihood estimates.

(c)- PARAM1 for the EXPONENTIAL distribution is the reciprocal of the sample mean. For the LOGNORMAL distribution PARAM1 and PARAM2 are μ and σ^2, the parameters of the lognormal distribution (equations (5) and (6) in Section IIA).

(d)- The MTTR and the 50th, 90th and 95th percentiles are calculated from the sample and are based on the relationships between the calculated parameters and their distribution functions (equations (7) and (10) in Section IIIA). The percentage error is between the exponential and lognormal MTTR and percentiles (equation (25) in Section VB).

(e)- The chi-square statistic is calculated from equation (12) - Section IIIB3.

(f)- The number of degrees of freedom is K-2 for the exponential distribution assumption and K-3 for the lognormal. This is because one parameter is estimated from the sample in the exponential case and two in the lognormal case.

(g)- The level of significance is the probability of a χ^2 variate with the specified degrees of freedom exceeding the calculated chi-square statistic.

The complete data, chi-square computation, and results of the above example are given in Appendix E.

2. Discussion of Results

The following discussion refers to those cases which either do not satisfy the underlying assumption or are of special interest as their results are different from the others and/or point out some interesting issues.

(a) Set No. 4 - AN/FPS-80

This case was presented as an unsuccessful one in the source paper [Ref. 15] due to inexperienced technicians and the need for adjustment factors to the repair times during the demonstration. That conclusion is based on the histogram which is presented in the paper which cannot be used to test a distributional assumption. Indeed, the results of the statistical tests and the plot on lognormal probability show that the lognormal distribution cannot be rejected with a high level of significance (0.92). The result of the chi-squared test for the exponential distribution shows that, had it been tested separately, one would fail to reject it with a level of significance of 0.15. Thus, one would tend to accept the lognormal distribution in this case due to the high level of significance as compared to the exponential. Therefore, in cases like this one, careful analysis must be made and more than a single test should be performed in order to determine the suitability of a distribution.

(b) Set No. 5 - AN/TPS-39(V)

This case was presented as a definite violation of the lognormal distribution characteristics. The reasons,

as given in the paper [Ref. 15], are attributed to the size of the equipment. It is suggested there that the normal distribution should be considered as an adequate descriptor because the repair times for small equipment are short and have small variations around a mean value. A histogram in the paper indicates "almost" a normal distribution. However, a chi-squared goodness-of-fit test for normality rejects this assumption at a 0.005 level of significance. The exponential and lognormal distribution assumptions are also rejected. One reason for this might be that the repair times are rounded-off to the nearest minute, and as a result there are clustered data points which do not follow any particular distribution. The log-normal probability plot (Figure 6) shows these clustered data points to which a straight line cannot be fitted. Other reasons behind this phenomenon, in this and other cases, require a separate analysis.

(c) Set No. 6 - AM/3949-GR

In this case, the histogram including all 57 repair times was bi-modal [Ref. 15] and, therefore, does not fit either distribution assumption. However, one third of the repair times were for a single fault, replacement of the transmitting tube. Filtering out these 19 data points resulted in a histogram in the paper which appears lognormal. Indeed, the statistical tests show this to be a valid assumption.

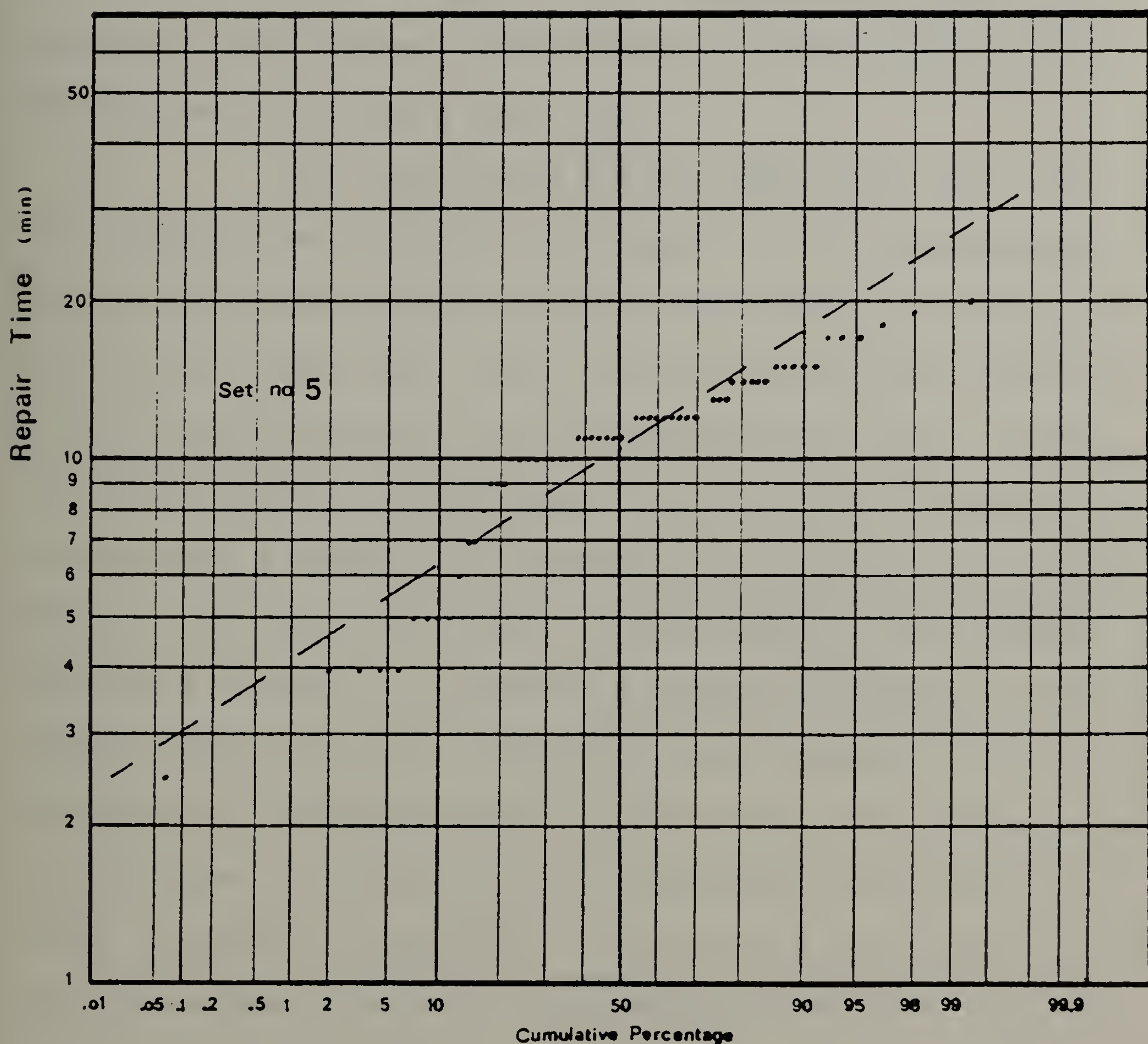


Figure 6: Lognormal Probability Plot [AN/TPS-39(V)]

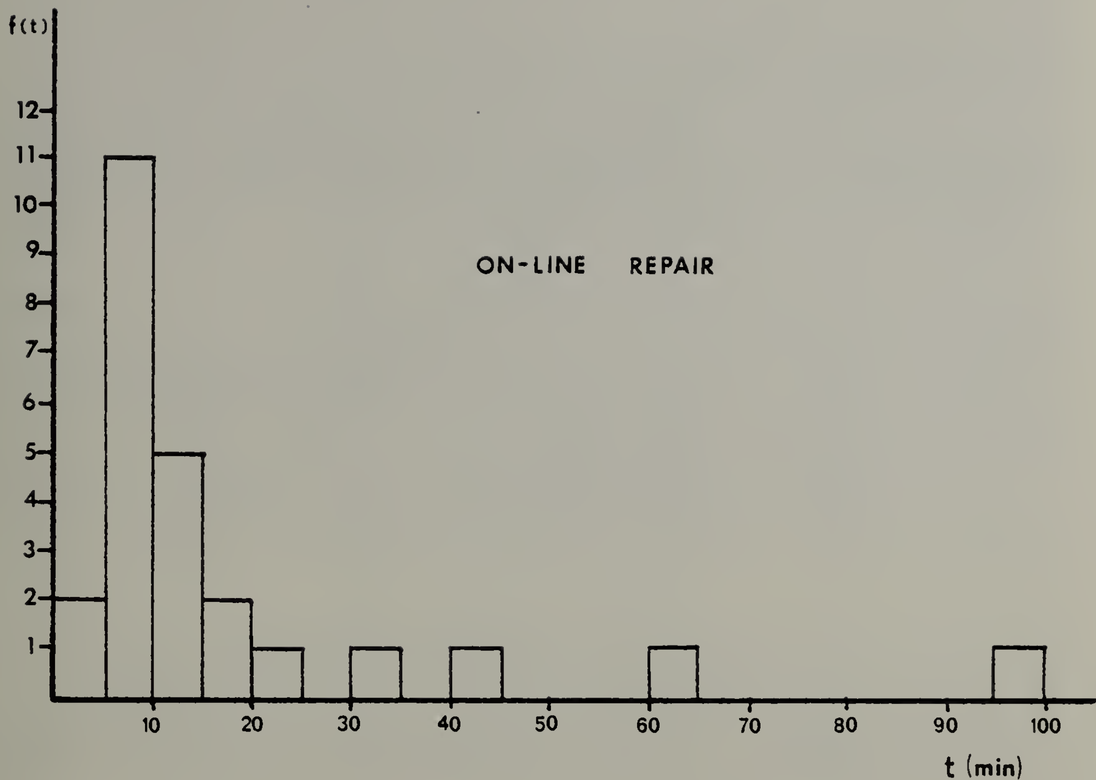
(d) Set No. 10 - DSIS-SCF

In this case, 50 tests were conducted during the maintainability demonstration, 25 for on-line repair and 25 for off-line repair. The equipment has much redundancy which allows on-line repair by "reconfiguring" the system by patching [Ref. 19].

For the on-line repair times (set 10a), the chi-squared test rejects the assumption for the exponential distribution but not for the lognormal. For the off-line repair times (set 10b), the chi-squared test resulted in the same probability for both assumptions and, in fact, neither assumption is rejected. A WE test for exponentiality and a W-test for lognormality resulted in "acceptance" of both distributions. A histogram of five minute intervals (Figure 7) indicates a roughly lognormal distribution which, together with the higher probability of getting the W test statistic and the lognormal probability plot (Figure 8), show that the lognormal distribution is still a "better" descriptor, and that off-line repair times are better described by a lognormal distribution than on-line repair times are.

(e) Sets No. 13, 14 and 18 - User Display Segments

The demonstration reports for these cases [Refs. 22, 23 and 27], include separate repair times - "inherent" and "achieved". The "achieved" repair time includes additional time required for obtaining test equipment, tools, spare items and maintainability information during the demonstration tests.



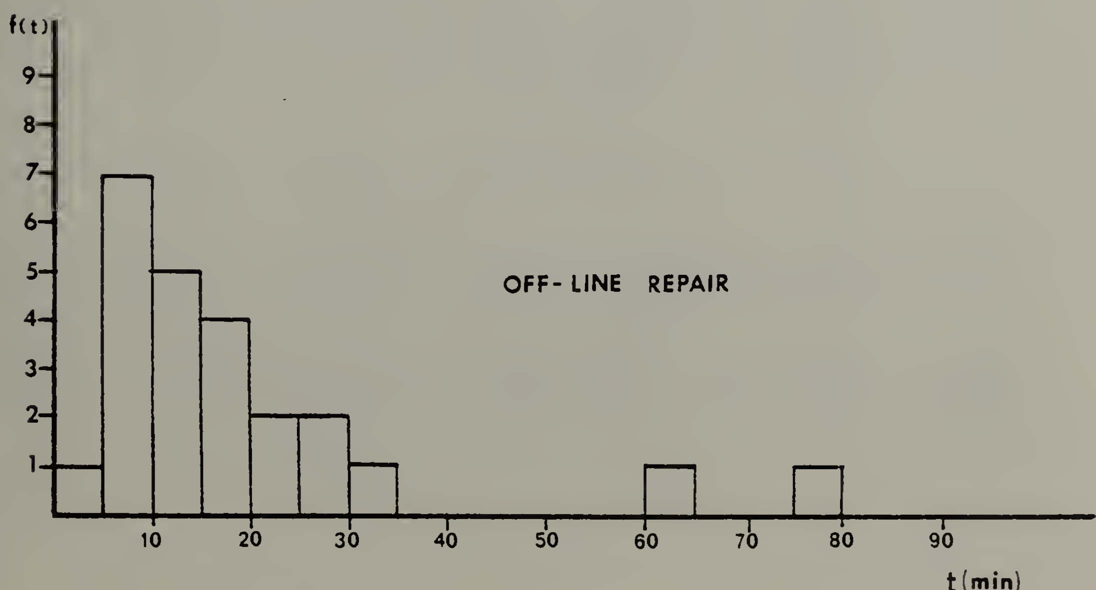


Figure 7: Five Minute Intervals Histogram [DSIS-SCF]

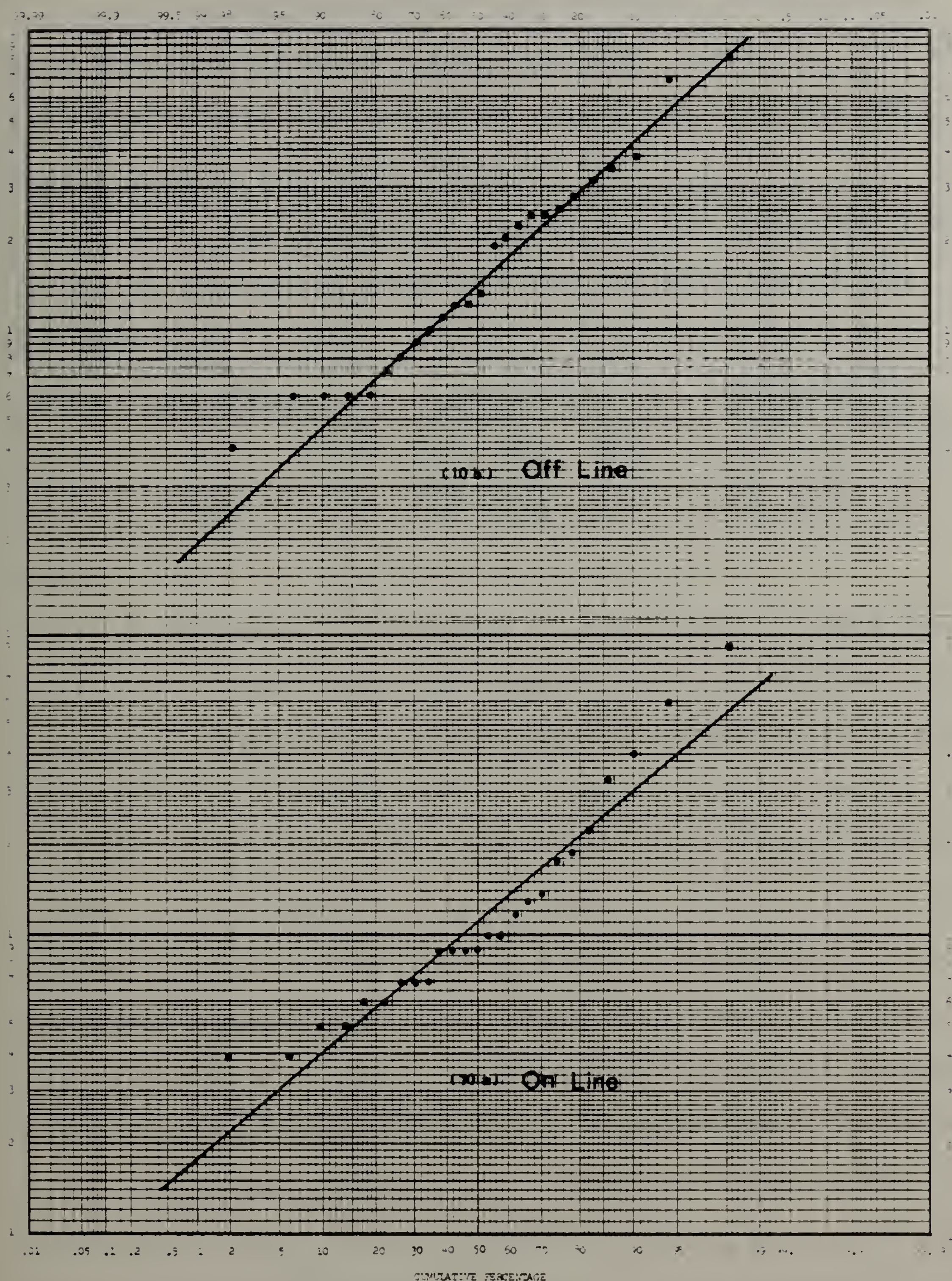


Figure 8: Lognormal Probability Plot [DSIS-SCF]

The validity of the lognormal distribution, when differentiating between "inherent" (sets 13a, 14a and 18a) repair time and "achieved" (sets 13b, 14b and 18b) repair time is not quite obvious.

The following results for the lognormal distribution assumption were obtained for each one of the six sets:

Case/Set	Inherent(a)			Achieved(b)		
Test:	χ^2	W	Plot	χ^2	W	Plot
13	N/R	N/R	G	R	N/R	P
14	N/R	R	P	N/R	N/R	G
18	R	N/R	G	N/R	N/R	G

where

N/R - Not-Rejected; R - Rejected; G - Good Fit; P - Poor Fit.

For "inherent" repair time the lognormal distribution assumption is not rejected by both tests in Case 13, but it is rejected by the W test in Case 14 and by the χ^2 test in Case 18. For "achieved" repair time, this assumption is rejected by the χ^2 test in Case 13, but it is not rejected by either test in Case 14 and Case 18. The lognormal probability plots (Appendix D) show poor fit in sets 13b and 14a. Since in these three cases the statistical tests indicate different results, no assessment can be made of the lognormal distribution assumption for "inherent" or "achieved" repair time. However, in sets 13a, 14b and 18b this assumption cannot be rejected. We were unable to account for these anomalies since no consistent pattern is evident. Further investigations and discussion with the individuals who conducted the demonstration tests is

required in order to account for this.

(f) <u>Sets No. 17 and 19 - HT/MT Terminal and Autodin Memory/Memory Control Equipment</u>

These two cases are discussed because their results would have been expected in the opposite way. While the analysis results indicate that the first case (Case 17) violates the assumption of lognormality and that in the second case (Case 19) the lognormal distribution fits the data, the assumptions made in the demonstration reports [Refs. 26 and 28] are that in the first one the lognormal distribution assumption is valid and in the second the exponential distribution is an adequate model.

The assumptions in the reports were based on the nature of the equipment, rather than on statistical tests. Probability plotting used in these reports have been found to be inaccurately interpreted (Case 17) and incorrect (Case 19).

In Case 17, the lognormal probability plot included in the report does not show a straight line; most of the deviations are at the lower level (Figure 9). But, it is concluded in the report [Ref. 26] that the lognormal distribution fits the data. Both the chi-squared and W tests reject this assumption. The assumption for the exponential distribution appears to be appropriate in this case. This is indicated by the chi-squared test for exponentiality, the exponential probability plot (Figure 10), and a histogram (Figure 11). The reasons for these results require further investigation and analysis.

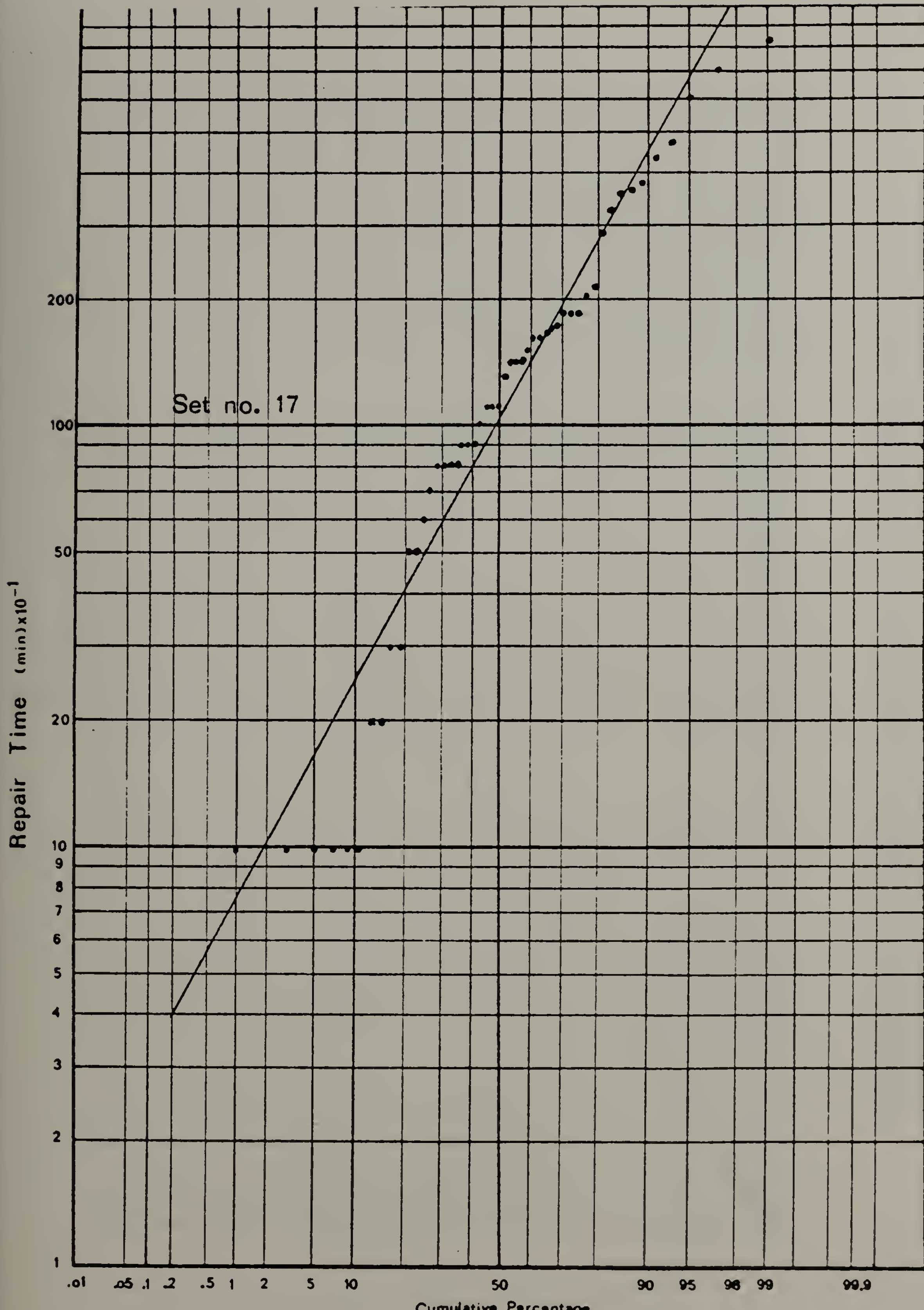


Figure 9: Lognormal Probability Plot [HT/MT Terminal]

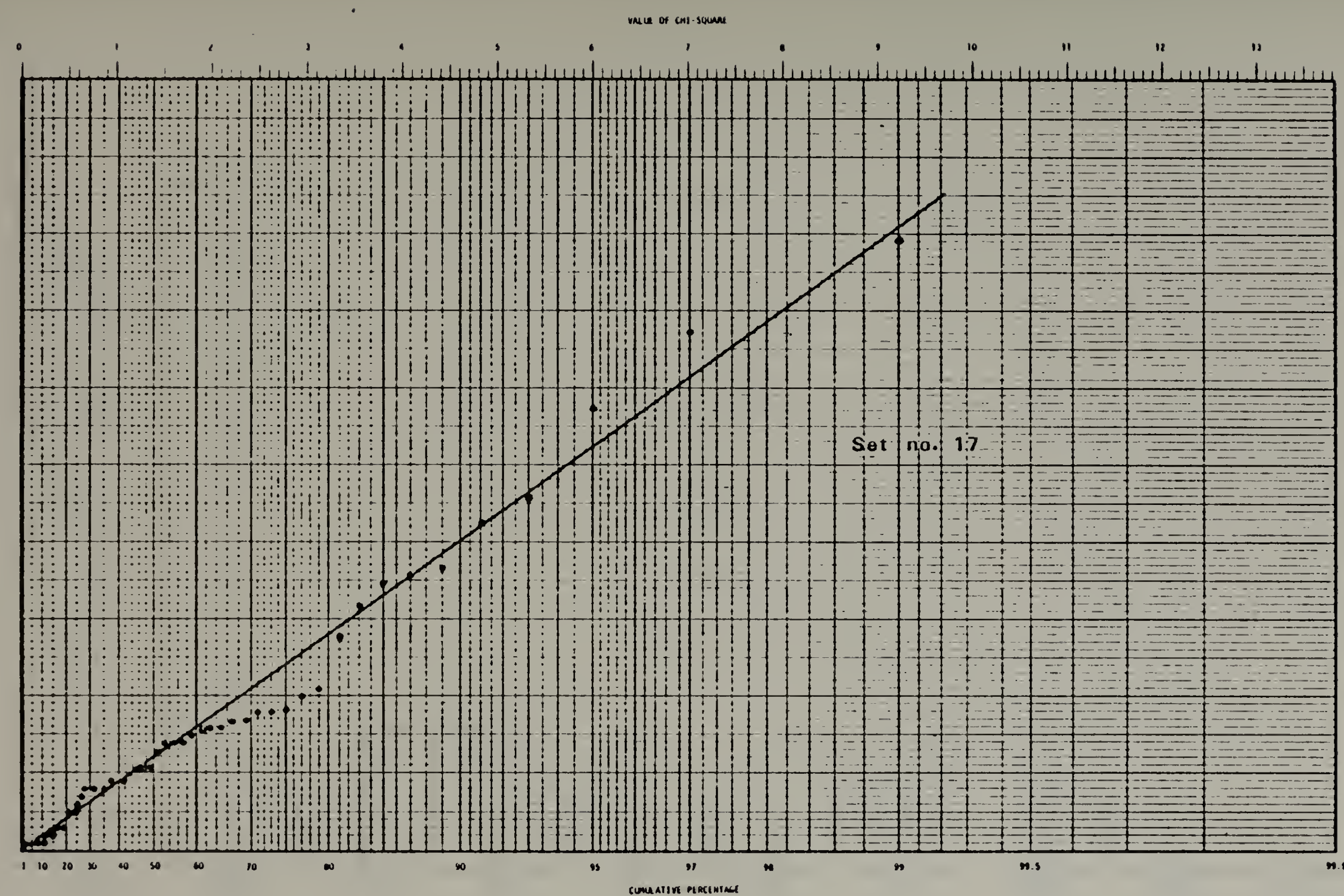
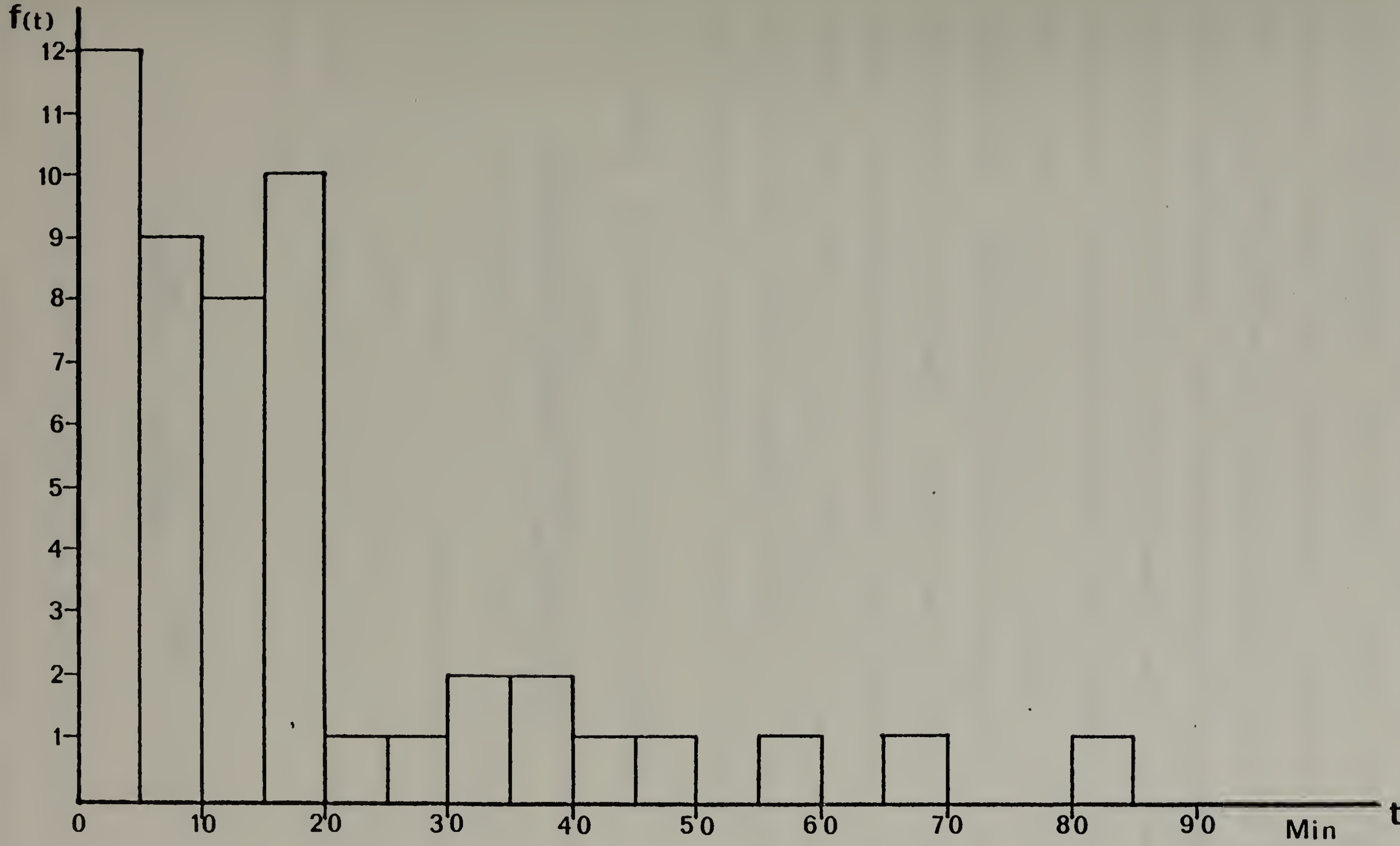


Figure 10: Exponential Probability Plot [HT/MT Terminal]



<u>Figure 11</u>: Five Minute Intervals Histogram [HT/MT Terminal]

In Case 19, a plot of the data on logarithmic graph paper included in the report "shows" a straight line. However, the way that plot was made is incorrect because the cumulative percentage points against which the data points were plotted, were calculated from the exponential distribution function using the sample mean, and not by calculating the expected values of the ordered observations. This is why the plot resulted in a straight line with no deviations. Multiple points are not taken into account when the plot is so made. Indeed the statistical test results, the lognormal probability plot (Figure 12), and the exponential probability plot (Figure 13) show that the exponential distribution assumption should be rejected, while the assumption of lognormality cannot be rejected.

B. ERRORS IN CALCULATED AND ESTIMATED PARAMETERS

1. Error in MTTR and Inherent Availability when Assuming an Exponential Distribution

The steady-state form of inherent availability, equation (1), is easily derived from calculus using assumptions of an exponential distribution for failure and repair times. The expression for availability as a function of time is then

$$A_i(t) = \frac{MTBF}{MTBF+MTTR} + \frac{MTTR}{MTBF+MTTR}\, e^{-\left[\frac{1}{MTTR} + \frac{1}{MTBF}\right] t} \qquad (19)$$

The steady-state term, the first term in the above equation, can be applied without making any assumptions on

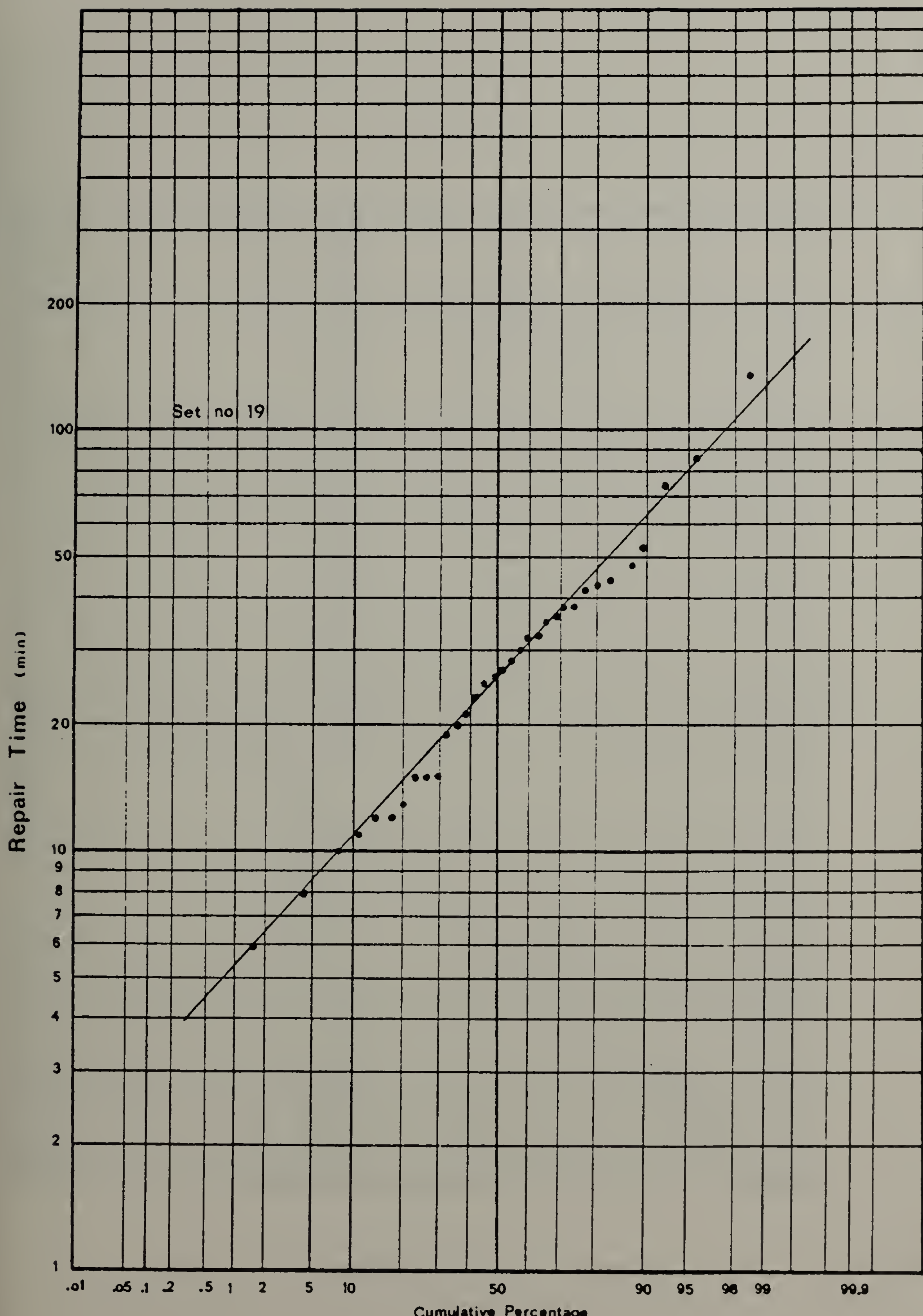


Figure 12: Lognormal Probability Plot [Autodin Memory/ Memory Control Equipment]

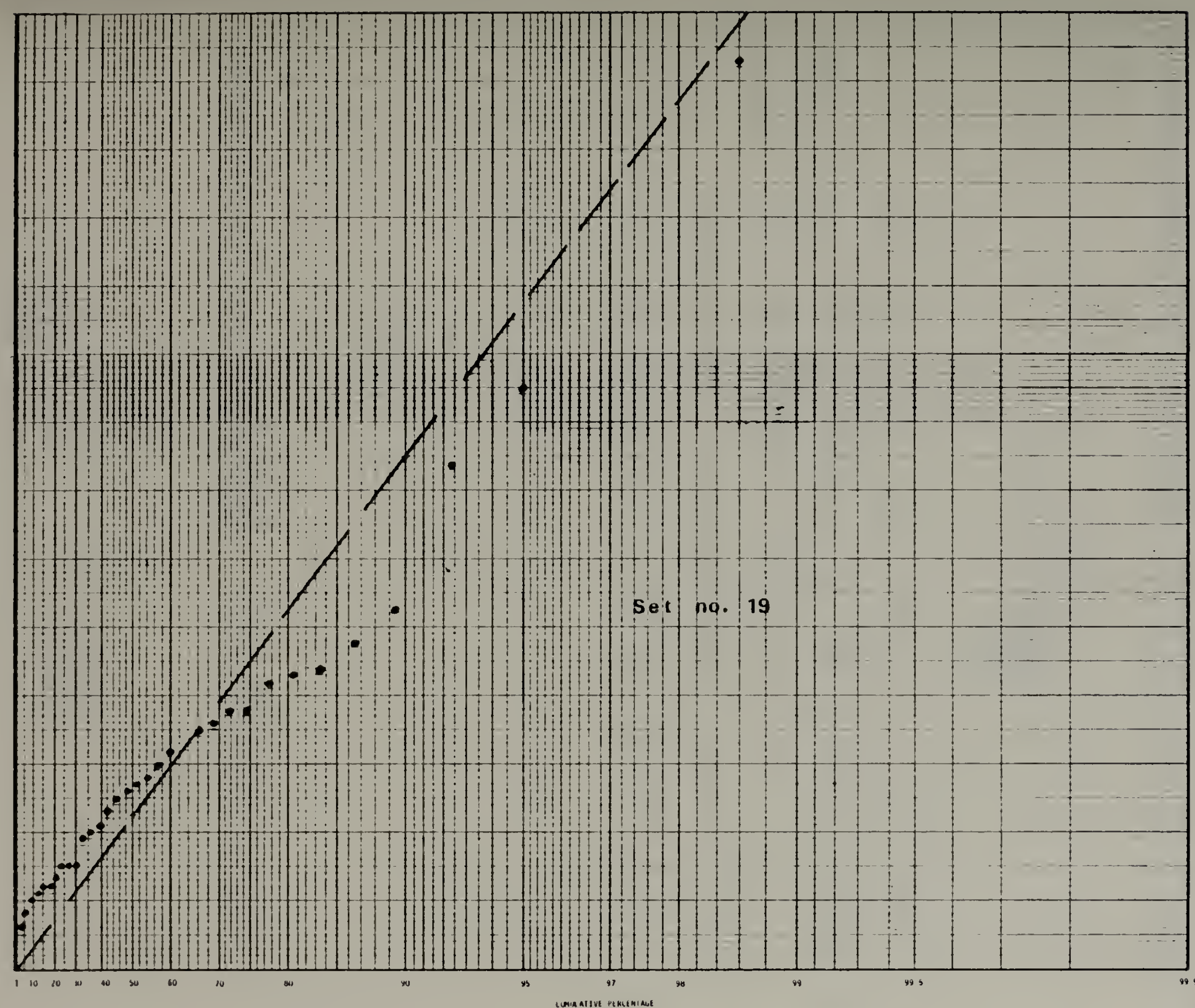


Figure 13: Exponential Probability Plot [Autodin Memory/ Memory Control Equipment]

the distributions. However, most theoretical papers and many applied papers are written using exponential distribution assumptions for both failure and repair times.

Since for a high availability what is desired is a high MTBF and a low MTTR, equation (1) can be rewritten as

$$A_i = \frac{1}{1 + \frac{MTTR}{MTBF}} \tag{20}$$

In a practical case, MTTR is of the order of one hour or less, while MTBF is of the order of 100 to 1000 hours. Thus, MTTR/MTBF ≈ 0.01 to 0.001.

Furthermore, the expression for availability (20) can be approximated from the series expansion of

$$\frac{1}{1+x} = 1 - x + x^2 - \ldots\ldots + (-1)^i x^i \tag{21}$$

$$= \sum_{i=0}^{\infty} (-1)^i x^i$$

Therefore, when $x = \frac{MTTR}{MTBF}$, this becomes

$$\frac{1}{1 + \frac{MTTR}{MTBF}} = \sum_{i=0}^{\infty} (-1)^i \left[\frac{MTTR}{MTBF}\right]^i \tag{22}$$

The approximation form is given by the first two terms of the expansion

$$\frac{MTBF}{MTBF+MTTR} \simeq 1 - \frac{MTTR}{MTBF} \tag{23}$$

The error in the approximation is less than the third term of the expansion

$$E < \left[\frac{MTTR}{MTBF}\right]^2 \tag{24}$$

Therefore, an error of few percent in MTTR by assuming an exponential distribution, instead of a lognormal distribution, will have negligible effect on availability, which is the measure of interest when dealing with operational readiness or system effectiveness.

The percentage error in the mean-time-to-repair is calculated as follows

$$E = \frac{|M_{LOG} - M_{EXP}|}{M_{LOG}} \times 100 \tag{25}$$

where

M_{LOG} = the lognormal mean = $e^{\mu+\frac{1}{2}\sigma^2}$, where μ and σ^2 are defined in equations (5) and (6) respectively

M_{EXP} = the exponential mean = $\bar{x}$, where $\bar{x}$ is the sample mean (equation (10))

The results of the percentage error in the mean, which are summarized in Table III, show that the error in MTTR is very small. All cases have an error less than 10% and the average error is less than 2.5%. The matter of

TABLE III

Percentage Error in MTTR when Assuming an Exponential Distribution

Set No.	Sample Mean $[M_{EXP}]$	M_{LGN}	E(%)
(1)	(2)	(3)	$(4) = \frac{\lvert(3)-(2)\rvert}{(3)}$
1	18.7	18.3	2.0
2	25.4	25.1	0.9
3	20.43	20.43	0.06
4	78.1	81.0	3.5
5	11.1	11.3	1.4
6	28.5	29.0	1.8
7	22.4	22.6	0.9
8a	20.2	20.3	0.6
8b	70.7	71.7	1.5
9	56.2	53.1	5.9
10a	17.4	16.4	6.2
10b	20.8	20.9	0.22
11	10.0	9.3	6.7
12	11.5	11.2	2.8
13a	48.2	48.3	0.24
13b	72.3	72.1	0.35
14a	50.4	54.7	7.8
14b	154.0	155.7	1.1
15	52.0	54.3	4.2
16	19.0	19.3	1.5
17	17.0	19.9	-
18a	41.6	42.9	2.9
18b	43.3	44.1	2.0
19	32.4	32.5	0.4

convenience in using the sample mean, instead of the log-normal mean, despite the error, is easily justified.

2. Percentage Error in Median and Upper Percentiles of the Lognormal Distribution

a. Error Caused when Assuming an Exponential Distribution

From Table IV, the average percentage error on the 50th, 90th and 95th percentiles, when calculated based on an exponential distribution instead of a lognormal distribution, is greater the higher the percentile. The average error for the median is 15% with a range from 3.7% to 61%. The average error for the 90th percentile is 21% with a range from 1.6% to 47%. The average error for the 95th percentile is 25% with a range from 0.7% to 65%.

The greater the probability of getting the test statistic for the lognormal distribution (which means more appropriateness of the lognormal distribution), the greater the error is when assuming an exponential distribution. This result is important in particular when estimating the median and maximum allowed corrective repair time during a maintainability demonstration.

b. Error Between Calculated and Estimated Parameters from Probability Plot

The percentage error when estimating parameters of the lognormal distribution from a lognormal probability plot instead of calculating them, is relatively small.

From Table V it can be seen that the percentage error in the median is less than 10% with an average of 3%.

TABLE IV

Percentage Error in Median and Upper Percentiles when Assuming an Exponential Distribution

Set No.	50th Percentile LGN	EXP	E(%)	90th Percentile LGN	EXP	E(%)	95th Percentile LGN	EXP	E(%)
1	11.4	12.9	13.8	39.7	43.0	8.3	56.6	55.9	1.2
2	20.2	17.6	13.1	47.0	58.4	24.0	59.7	75.9	27.0
3	15.3	14.2	7.6	40.5	47.0	16.3	53.3	61.0	15.0
4	49.0	54.2	10.5	177.1	179.9	1.6	254.7	234.1	8.1
5	10.4	7.7	26.0	17.5	25.6	47.0	20.2	33.3	65.0
6	25.6	19.8	23.0	48.5	65.6	35.0	58.1	85.4	47.0
7	19.6	15.6	21.0	39.1	51.6	32.0	47.5	67.2	41.0
8a	16.7	14.0	16.0	37.3	46.5	25.0	46.8	60.6	29.0
8b	62.3	49.0	21.0	123.0	162.7	32.0	149.0	211.7	42.0
9	24.3	39.0	61.0	120.8	129.5	7.3	190.2	168.5	11.4
10a	11.6	12.1	3.7	33.6	40.1	19.3	45.3	52.1	15.0
10b	15.2	14.4	5.2	42.2	48.0	13.8	56.3	62.4	10.9
11	7.6	6.9	.9.8	17.2	23.0	34.0	21.6	29.9	38.0
12	9.1	8.0	12.1	20.9	26.6	27.0	26.4	34.6	31.0
13a	38.0	33.4	12.0	92.4	110.9	20.0	118.8	144.3	21.0
13b	63.0	50.1	20.0	122.0	166.0	36.0	148.0	216.0	46.0
14a	37.1	34.9	5.8	114.7	116.1	1.2	157.9	151.0	4.4
14b	118.1	106.7	9.6	306.0	355.0	15.8	401.0	461.0	15.0
15	37.5	36.1	3.7	113.2	119.8	5.9	154.8	155.9	0.7
16	17.0	13.2	22.0	32.4	43.8	35.0	38.9	56.9	46.0
17	19.9	17.0	--	45.0	39.2	--	69.0	51.0	--
18a	30.8	28.9	6.2	87.4	95.9	9.7	117.4	124.7	6.2
18b	32.4	30.0	7.3	88.9	99.7	12.1	118.3	129.7	9.6
19	25.6	22.5	12.2	62.2	74.6	19.9	80.0	97.0	21.0

TABLE V

Percentage Error between Calculated Lognormal Percentile and Estimated from Plot

Set No.	50th Percentile			90th Percentile			95th Percentile		
	Calculated	Estimated	E(%)	Calculated	Estimated	E(%)	Calculated	Estimated	E(%)
1	11.4	11.8	3.5	39.7	40.0	0.8	56.6	57.0	0.7
2	20.2	18.5	8.4	47.0	49.0	4.2	59.7	65.0	8.9
3	15.3	15.5	1.3	40.5	40.0	1.2	53.3	51.0	4.3
4	49.0	50.0	2.0	177.1	180.0	1.7	254.7	260.0	2.1
5	10.4	--	--	17.5	--	--	20.2	--	--
6	25.6	25.0	2.3	48.5	46.0	5.1	58.1	55.0	5.3
7	19.6	20.5	4.6	39.1	45.0	15.1	47.5	55.0	15.8
8a	16.7	17.0	1.8	37.3	38.0	1.9	46.8	47.0	0.4
8b	62.3	63.0	1.1	123.0	122.0	0.8	149.0	150.0	0.7
9	24.3	22.0	9.5	120.8	100.0	17.2	190.2	150.0	20.4
10a	11.6	11.0	5.2	33.6	30.0	10.7	45.3	40.0	11.7
10b	15.2	14.0	7.9	42.2	42.0	0.5	56.3	58.0	3.0
11	7.6	--	--	17.2	--	--	21.6	--	--
12	9.1	9.3	2.2	20.9	21.0	0.5	26.4	26.5	0.4
13a	38.0	37.0	2.6	92.4	88.0	4.8	118.0	120.0	1.7
13b	63.0	63.0	0.0	122.0	122.0	0.0	148.0	150.0	1.4
14a	37.1	39.0	5.1	114.7	145.0	26.4	157.9	210.0	33.0
14b	118.1	115.0	2.6	306.0	330.0	7.8	401.0	450.0	12.2
15	37.5	36.0	4.0	113.2	108.0	4.6	154.8	145.0	6.3
16	17.0	17.0	0.0	32.4	31.5	2.8	38.9	37.0	4.9
17	19.9	--	--	45.0	--	--	69.0	--	--
18a	30.8	30.0	2.6	87.4	82.5	5.6	117.4	112.0	4.6
18b	32.4	32.0	1.2	88.9	90.0	1.2	118.3	120.0	1.4
19	25.6	26.0	1.6	62.2	63.0	1.3	80.0	81.0	1.3

In the upper percentiles the percentage error is higher, but still within a reasonable range. In one third of the cases the error is more than 10% and this is particularly so in which the lognormal distribution does not fit the data.

From these results it can be concluded that the estimation of distribution parameters from a probability plot is a convenient and sufficiently accurate method, depending on the purpose and the intended use of such estimated parameters.

C. EXTREME VALUE POINTS IN PROBABILITY PLOTTING

Most of the plots on lognormal probability paper resulted in some departures from the straight line which was drawn as a linear fit to the plotted data points, especially at the extreme values. While deviations in the central part of the line (between the first and third quartiles), can be attributed to randomness and inaccuracy (round-offs) of measurements and plotting, the deviations of the extreme value points require some additional explanation.

The greatest expected deviations are those points at the higher level which end up below the straight line. One reason for this is truncation of tests once the repair time becomes too long during the demonstration test, and as a result the repair time is estimated. On the other hand, in the field it can be expected that the actual repair time would more closely follow the pattern of the line or lie above it because of conditions in the field.

The reasons for the deviations of the lower extreme value points below the line are probably imprecise measurements of short times.

The deviations above the line might be explained as special cases in which there are inexperienced technicians during the demonstration and as a result some repair times become longer than expected.

Although these deviations are taken into account while fitting a straight line to plotted data points, they have more significant effect on the results of the tests for distributional assumptions. In some cases extreme points, usually those which were of totally different magnitude from the rest of the data points, were removed in order to determine their effect on the results. Indeed, some of these cases resulted in a "better" test statistic. However, the results of the statistical tests are based on the original data, including extreme value points, except for some cases discussed in Section VA2.

VI. CONCLUSIONS

A. SUMMARY AND CONCLUSIONS

The conclusions derived from this study may be divided into (1) conclusions concerning the results of the data analysis, and (2) conclusions related to the methods used for testing and analyzing the data and their results.

1. From the data analysis conducted in this study, it is concluded that the lognormal distribution is a good descriptor of the distribution of corrective maintenance repair time. Sixteen of the 24 sets from maintainability demonstrations of radically different designs tend to show that, within an acceptable level of significance, this assumption cannot be rejected. Similarly, the data analysis shows that the assumption of an exponential distribution should be rejected in 17 sets.

2. The percentage error in the MTTR when, assuming an exponential distribution instead of a lognormal distribution, as a matter of convenience, for calculating system availability has been found to be small. Other than the one case in which the exponential distribution assumption would not be rejected and the lognormal distribution assumption would, all cases have an error less than 10% and thus would not have any significant effect on availability.

3. The methods used in the analysis, probability plotting and statistical tests for distributional assumption,

complement one another. When the results of the statistical test indicated opposite conclusions, the probability plot or a histogram were helpful in determining the "correct" conclusion. Since there are differences among the sets of data and their accuracy, a single method of analysis is usually not sufficient.

4. The differences in the level of significance which resulted from the chi-squared test and the W test, can be attributed to the difference in computing the test statistics in each test. Points from the sample data which do not follow the assumed distribution will have different effects on the chi-squared test and the W test. The result of the test is also a function of both the mean and standard deviation estimated from the sample because these values are used in the calculation of the test statistic, although differently for each test.

5. Histograms, frequently used by some investigators, were found to be helpful when the results of the statistical tests were close. But in a histogram there is often a loss of information. It can be used only when there is a need to get insight on the shape of the distribution of the data.

6. Probability plots were found to be very useful in determining the suitability of a particular distribution and estimating its percentiles, and sometimes density parameters. They might be considered old fashioned in today's automated and computerized world. But, it is a very quick

and simple technique, which in addition to or in place of numerical methods of data analysis, can serve several purposes. In maintainability prediction and demonstration, the value of plotted data is quite significant. Estimation of distribution percentiles and parameters is easily obtained from the straight line drawn on the plot. The average error in doing so is very small. Non-random departures of the plotted data from a straight line can provide useful engineering information. Such departures may indicate the inadequacy of an assumed model, which implies that the parameters required to be tested might be wrong. It also may indicate that certain data points, such as extreme value points, do not follow the pattern of the rest of the data. Engineering insight can be gained when the reason for such deviations is determined.

B. RECOMMENDATIONS FOR CONTINUED RESEARCH

As regards continued research, five areas of special interest are recommended. The first one is the determination of expected ranges of the mean time to repair and other principal distribution parameters for different classes of equipment. Although this was one of the objectives of this study, because of the small number of data sets obtained for similar systems and the need for further investigation of the nature of the systems, it was not possible to make such a determination at this stage.

The second area is related to the difference between the chi-squared goodness-of-fit test and the W-test, from a

theoretical standpoint. The reasons behind such differences, which sometimes gave opposite results, should be investigated in order to determine the appropriateness of each test method in different cases.

The third area is also related to the results of this study. It is the investigation of those cases in which the lognormal distribution was rejected in order to discover the underlying reasons therefore.

The last two recommended areas are related to different type of systems. Namely, the fourth area is mechanical equipment repair times, an area in which maintainability demonstration data were not obtainable. In this case the remove/replace or repair actions may be of significantly larger magnitude than diagnostic time, and the lognormal distribution assumption may not be valid. The last area is related to the increasing use of digital techniques in electronic equipment with increasing use of automatic fault detection and built-in test. Coupled with the increasing use of microelectronics, the reduction in diagnostic and repair times may show MTTR's of smaller magnitude that appear to be exponential due to the limitations in taking small time observations. Here, again, the validity of the lognormal assumption should be verified.

APPENDIX A

MAINTAINABILITY DEMONSTRATION TEST METHODS
(Methods 4,8,9 in Appendix B of MIL-STD-471A)

1. General

Appendix B of MIL-STD-471A [Ref. 3] contains test methods and criteria for demonstrating the achievement of specified quantitative maintainability requirements.

2. Application

Table VI summarizes the major characteristics of the most used test methods for the median or mean-time-to-repair and the allowed maximum repair time (usually the 90th or 95th percentile of the distribution). Each test method provides an equation or other directions for determining a minimum sample size of maintenance tasks and it also provides decision criteria for acceptance or rejection of the item being demonstrated.

3. Test Methods

The concept of maintainability demonstration is based on the assumption that a sample of maintenance tasks corresponds to those expected in the field during the operating life of the system and can be used to make an assessment from the parameters measured in the sample.

This sample must be obtained in accordance with test procedures designed to ensure that the measures obtained are representative of the stated population of maintenance tasks, that the variables are adequately described and their units

TABLE VI

Characteristics of Test Methods in Maintainability Demonstration

	Test Method 4	Test Method 8	Test Method 9
Test Index	Median	Mean and 90th or 95th Percentile	Mean (corrective task time; preventive maintenance time; downtime) - M_{max}
Test Type and Theory	Test for the Median	Sequential Analysis	Central limit theorem
Assumptions (Distribution)	-A specific variance (σ = 0.55) -Lognormal distribution	Lognormal Distribution	Lognormal distribution for corrective task time
Sample Size	Minimum 20	See test method	Minimum 30
Sample Selection	Natural occurring failures or Appendix A	Natural occurring failures or simple	Natural occurring failures or Appendix A
Specified Requirements	-ERT (Equipment Repair Time)	-Mean -M_{max}	-μ_c, μ_{pm}, $\mu_{p/c}$

of measurements are independent, and that the test plan is sufficiently flexible to encompass variations in test conditions and schedules and is realistic in terms of existing constraints and the capability of test personnel [Ref. 8].

Appendix A of MIL-STD-471A outlines a procedure for the selection of a sample of corrective maintenance tasks for maintainability demonstration when the tasks result from failure simulation. The objectives of this procedure are to allow for the selection of maintenance tasks such that the selection simulates the failure frequency of the test unit in actual operation, and to insure that a proportionately representative sample of task types/times are selected. The sequential test method (Test Method 8) employs simple random sampling.

The following is a brief description of the test methods.

(a) <u>Test Method 4 - Test on the Median (ERT)</u>
<u>(Test Method 3 in MIL-STD-471)</u>

This method is used when the requirement is stated in terms of an Equipment Repair Time (ERT), which is the median specified in the detailed equipment specification.

The decision rule states that the equipment under test is considered to have met the required ERT when the measured mean-time-to-repair ($MTTR_G$) and standard deviation (S), as determined in Appendix B of MIL-STD-471A, satisfy the expression

$$\text{Log } MTTR_G \leq \text{Log } ERT + 0.397(S)$$

The specified ERT in the equipment specification should be determined using the expression

$$\text{ERT (specified)} = 0.37\ \text{ERT}_{max}$$

where

ERT_{max} = the maximum value of ERT that should be accepted no more than ten percent of the time.

0.37 = a value resulting from application of "student's t" operating characteristics for a sample size of 20 at a five percent level of significance and assuming a population standard deviation of 0.55.

(a probability of 0.05 of rejecting a system having a true MTTR_G equal to the specified ERT as a result of one test).

(b) <u>Test Method 8 - Test on a Combined Mean/Percentile Requirement</u> (Test Method 1 in MIL-STD-471)

This method is used when the specifications are in terms of a dual requirement for the mean and either the 90th or 95th percentile of maintenance times when the distribution is lognormal.

It is assumed that the mean is greater than 100 units of time, the ratio of the 90th percentile to the mean is less than two, and the ratio of the 95th percentile to the mean is less than three.

The accept/reject criteria for the values of the required mean, M_{max} (90th or 95th percentile) are defined

in three separate plans/tables. The number of observations greater than and less than the required value are accumulated separately and compared to the decision values shown in the tables. For example, for a sample size of 50 the accept criterion is if the number of observations less than the specified mean is 11 and less than the specified 90th percentile is one. The reject criterion is if the number of observations greater than the specified mean is 19 and greater than the specified M_{max} is four for the 90th percentile or three for the 95th percentile.

When an accept decision for one of the parameters is reached, only the test for the second parameter should continue. The equipment is rejected when a decision to reject either parameter occurs regardless of the status of the other parameter.

If no accept or reject decision is made after 100 observations, the following rule applies:

Mean - Accept only if 29 or less observations are more than the value of the required mean.

M_{max} (90th percentile) - Accept only if 5 or less observations are more than M_{max} specified.

M_{max} (95th percentile) - Accept only if 2 or less observations are more than M_{max} specified.

(c) <u>Test Method 9 - Test for Mean Maintenance Time (corrective, preventive, and combination of corrective and preventive) and M_{max}</u> (Test Method 2 in MIL-STD-471)

This method is applicable to demonstration of Mean Corrective Maintenance Time (μ_c), Mean Preventive

Maintenance Time (μ_{pm}), Mean Maintenance Time (includes preventive and corrective maintenance actions)($\mu_{p/c}$), and M_{max} (90th or 95th percentile of the repair time).

The procedures of this method for demonstrating μ_c are based on the Central Limit Theorem. The minimum sample size is 30, but the actual sample size should be determined for each equipment demonstrated. The procedure for demonstrating M_{max} is valid for those cases where the underlying distribution of corrective maintenance times is lognormal.

The accept/reject criteria are one tailed confidence levels for specified level of consumer risk.

APPENDIX B

THE EXPECTED VALUE OF ORDERED OBSERVATIONS FOR PROBABILITY PLOTTING

The principle of probability plotting requires the plotting of the ordered observations versus their "expected values". Probability papers are designed such that the ordered observed values, when plotted against their expected values, would lie on an approximate straight line through the origin with slope equals to one. The origin and slope of the plot will change if the variable is linearly transformed for plotting convenience, but the plot will still result in a straight line [Ref. 12].

Let f(x) be the probability density function and F(x) be the cumulative distribution function of a population from which a large number of samples of size n are selected.

Let x_i be the value of the i-th smallest observation in a particular sample.

Because x_i is a random variable, its value fluctuates from one sample to the next according to some probability distribution whose expected value $E(x_{i,n})$, can be shown to be equal to [Ref. 4]:

$$E(x_{i,n}) = \frac{n!}{(i-1)!(n-1)!} \int_0^1 x_i [F(x_i)]^{i-1} [1-F(x_i)]^{n-i} \, d[F(x_i)]$$

where

$i=1,2,\ldots,n; \; -\infty < x_i < \infty$ and $x_i \le x_{i+1}$ for all i.

For distributions for which $E(x_{i,n})$ cannot be determined exactly, the following approximation can be used:

$$E(x_{i,n}) = F^{-1}\left[\frac{i-c}{n-2c+1}\right], \qquad i=1,2,\ldots,n$$

where

$F^{-1}\left[\frac{i-c}{n-2c+1}\right]$ is the value of x, such that

$$F(x) = \int_{-\infty}^{x} f(y)dy = \frac{i-c}{n-2c+1}$$

Thus, $E(x_{i,n})$ is the F(x)-th fractile of the distribution, and c is a number which depends on n and f(x) [Ref. 4].

The value of $c = \frac{1}{2}$, as suggested in References 4 and 11, was used in this study. This value of c has been found generally acceptable for a wide variety of distributions and sample sizes. However, the use of c=0 has also been suggested in some related works [Ref. 12].

APPENDIX C

TABLES FOR EVALUATING W TEST FOR NORMALITY (*)

TABLE VII

Coefficients Used in W Test for Normality

i \ n	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	0.7071	0.6872	0.6646	0.6431	0.6233	0.6052	0.5888	0.5739	0.5601	0.5475	0.5359	0.5251	0.5150	0.5056	0.4968	0.4886
2		0.1677	0.2413	0.2806	0.3031	0.3164	0.3244	0.3291	0.3315	0.3325	0.3325	0.3318	0.3306	0.3290	0.3273	0.3253
3				0.0875	0.1401	0.1743	0.1976	0.2141	0.2260	0.2347	0.2412	0.2460	0.2495	0.2521	0.2540	0.2553
4						0.0561	0.0947	0.1224	0.1429	0.1586	0.1707	0.1802	0.1878	0.1939	0.1988	0.2027
5								0.0399	0.0695	0.0922	0.1099	0.1240	0.1353	0.1447	0.1524	0.1587
6										0.0303	0.0539	0.0727	0.0880	0.1005	0.1109	0.1197
7												0.0240	0.0433	0.0593	0.0725	0.0837
8														0.0196	0.0359	0.0496
9																0.0163

i \ n	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34
1	0.4808	0.4734	0.4643	0.4590	0.4542	0.4493	0.4450	0.4407	0.4366	0.4328	0.4291	0.4254	0.4220	0.4188	0.4156	0.4127
2	0.3232	0.3211	0.3185	0.3156	0.3126	0.3098	0.3069	0.3043	0.3018	0.2992	0.2968	0.2944	0.2921	0.2398	0.2876	0.2854
3	0.2561	0.2565	0.2578	0.2571	0.2563	0.2554	0.2543	0.2533	0.2522	0.2510	0.2499	0.2487	0.2475	0.2463	0.2451	0.2439
4	0.2059	0.2085	0.2119	0.2131	0.2139	0.2145	0.2148	0.2151	0.2152	0.2151	0.2150	0.2148	0.2145	0.2141	0.2137	0.2132
5	0.1641	0.1686	0.1736	0.1764	0.1787	0.1807	0.1822	0.1836	0.1848	0.1857	0.1864	0.1870	0.1874	0.1878	0.1880	0.1882
6	0.1271	0.1334	0.1399	0.1443	0.1480	0.1512	0.1539	0.1563	0.1584	0.1601	0.1616	0.1630	0.1641	0.1651	0.1660	0.1667
7	0.0932	0.1013	0.1092	0.1150	0.1201	0.1245	0.1283	0.1316	0.1346	0.1372	0.1395	0.1415	0.1433	0.1449	0.1463	0.1475
8	0.0612	0.0711	0.0804	0.0878	0.0941	0.0997	0.1046	0.1089	0.1128	0.1162	0.1192	0.1219	0.1243	0.1265	0.1284	0.1301
9	0.0303	0.0422	0.0530	0.0618	0.0696	0.0764	0.0823	0.0876	0.0923	0.0965	0.1002	0.1036	0.1066	0.1093	0.1118	0.1140
10		0.0140	0.0263	0.0368	0.0459	0.0539	0.0610	0.0672	0.0728	0.0778	0.0822	0.0862	0.0899	0.0931	0.0961	0.0988
11				0.0122	0.0228	0.0321	0.0403	0.0476	0.0540	0.0598	0.0650	0.0697	0.0739	0.0777	0.0812	0.0844
12						0.0107	0.0200	0.0284	0.0358	0.0424	0.0483	0.0537	0.0585	0.0629	0.0669	0.0706
13								0.0094	0.0178	0.0253	0.0320	0.0381	0.0435	0.0485	0.0530	0.0572
14										0.0084	0.0159	0.0227	0.0289	0.0344	0.0395	0.0441
15												0.0076	0.0144	0.0206	0.0262	0.0314
16														0.0068	0.0131	0.0187
17																0.0062

(*) From Hahn, G. J. and Shapiro S. S., Statistical Models in Engineering, John Wiley and Sons, Inc., New York, 1967.

TABLE VII (Continued)

i \ n	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	0.4096	0.4068	0.4040	0.4015	0.3989	0.3964	0.3940	0.3917	0.3894	0.3872	0.3850	0.3830	0.3808	0.3789	0.3770	0.3751
2	0.2834	0.2813	0.2794	0.2774	0.2755	0.2737	0.2719	0.2701	0.2684	0.2667	0.2651	0.2635	0.2620	0.2604	0.2589	0.2574
3	0.2427	0.2415	0.2403	0.2391	0.2380	0.2368	0.2357	0.2345	0.2334	0.2323	0.2313	0.2302	0.2291	0.2281	0.2271	0.2260
4	0.2127	0.2121	0.2116	0.2110	0.2104	0.2098	0.2091	0.2085	0.2078	0.2072	0.2065	0.2058	0.2052	0.2045	0.2038	0.2032
5	0.1883	0.1883	0.1883	0.1881	0.1880	0.1878	0.1876	0.1874	0.1871	0.1868	0.1865	0.1862	0.1859	0.1855	0.1851	0.1847
6	0.1673	0.1678	0.1683	0.1686	0.1689	0.1691	0.1693	0.1694	0.1695	0.1695	0.1695	0.1695	0.1695	0.1693	0.1692	0.1691
7	0.1487	0.1496	0.1505	0.1513	0.1520	0.1526	0.1531	0.1535	0.1539	0.1542	0.1545	0.1548	0.1550	0.1551	0.1553	0.1554
8	0.1317	0.1331	0.1344	0.1356	0.1366	0.1376	0.1384	0.1392	0.1398	0.1405	0.1410	0.1415	0.1420	0.1423	0.1427	0.1430
9	0.1160	0.1179	0.1196	0.1211	0.1225	0.1237	0.1249	0.1259	0.1269	0.1278	0.1286	0.1293	0.1300	0.1306	0.1312	0.1317
10	0.1013	0.1036	0.1056	0.1075	0.1092	0.1108	0.1123	0.1136	0.1149	0.1160	0.1170	0.1180	0.1189	0.1197	0.1205	0.1212
11	0.0873	0.0900	0.0924	0.0947	0.0967	0.0986	0.1004	0.1020	0.1035	0.1049	0.1062	0.1073	0.1085	0.1095	0.1105	0.1113
12	0.0739	0.0770	0.0798	0.0824	0.0848	0.0870	0.0891	0.0909	0.0927	0.0943	0.0959	0.0972	0.0986	0.0998	0.1010	0.1020
13	0.0610	0.0645	0.0677	0.0706	0.0733	0.0759	0.0782	0.0804	0.0824	0.0842	0.0860	0.0876	0.0892	0.0906	0.0919	0.0932
14	0.0484	0.0523	0.0559	0.0592	0.0622	0.0651	0.0677	0.0701	0.0724	0.0745	0.0765	0.0783	0.0801	0.0817	0.0832	0.0846
15	0.0361	0.0404	0.0444	0.0481	0.0515	0.0546	0.0575	0.0602	0.0628	0.0651	0.0673	0.0694	0.0713	0.0731	0.0748	0.0764
16	0.0239	0.0287	0.0331	0.0372	0.0409	0.0444	0.0476	0.0506	0.0534	0.0560	0.0584	0.0607	0.0628	0.0648	0.0667	0.0685
17	0.0119	0.0172	0.0220	0.0264	0.0305	0.0343	0.0379	0.0411	0.0442	0.0471	0.0497	0.0522	0.0546	0.0568	0.0588	0.0608
18		0.0057	0.0110	0.0158	0.0203	0.0244	0.0283	0.0318	0.0352	0.0383	0.0412	0.0439	0.0465	0.0489	0.0511	0.0532
19				0.0053	0.0101	0.0146	0.0188	0.0227	0.0263	0.0296	0.0328	0.0357	0.0385	0.0411	0.0436	0.0459
20						0.0049	0.0094	0.0136	0.0175	0.0211	0.0245	0.0277	0.0307	0.0335	0.0361	0.0386
21								0.0045	0.0087	0.0126	0.0163	0.0197	0.0229	0.0259	0.0288	0.0314
22										0.0042	0.0081	0.0118	0.0153	0.0185	0.0215	0.0244
23												0.0039	0.0076	0.0111	0.0143	0.0174
24														0.0037	0.0071	0.0104
25																0.0035

TABLE VIII

Percentage Points of W Test for Normality

n	1	2	5	10	50
3	0.753	0.756	0.767	0.789	0.959
4	0.687	0.707	0.748	0.792	0.935
5	0.686	0.715	0.762	0.806	0.927
6	0.713	0.743	0.788	0.826	0.927
7	0.730	0.760	0.803	0.838	0.928
8	0.749	0.778	0.818	0.851	0.932
9	0.764	0.791	0.829	0.859	0.935
10	0.781	0.806	0.842	0.869	0.938
11	0.792	0.817	0.850	0.876	0.940
12	0.805	0.828	0.859	0.883	0.943
13	0.814	0.837	0.866	0.889	0.945
14	0.825	0.846	0.874	0.895	0.947
15	0.835	0.855	0.881	0.901	0.950
16	0.844	0.863	0.887	0.906	0.952
17	0.851	0.869	0.892	0.910	0.954
18	0.858	0.874	0.897	0.914	0.956
19	0.863	0.879	0.901	0.917	0.957
20	0.868	0.884	0.905	0.920	0.959
21	0.873	0.888	0.908	0.923	0.960
22	0.878	0.892	0.911	0.926	0.961
23	0.881	0.895	0.914	0.928	0.962
24	0.884	0.898	0.916	0.930	0.963
25	0.888	0.901	0.918	0.931	0.964
26	0.891	0.904	0.920	0.933	0.965
27	0.894	0.906	0.923	0.935	0.965
28	0.896	0.908	0.924	0.936	0.966
29	0.898	0.910	0.926	0.937	0.966
30	0.900	0.912	0.927	0.939	0.967
31	0.902	0.914	0.929	0.940	0.967
32	0.904	0.915	0.930	0.941	0.968
33	0.906	0.917	0.931	0.942	0.968
34	0.908	0.919	0.933	0.943	0.969
35	0.910	0.920	0.934	0.944	0.969
36	0.912	0.922	0.935	0.945	0.970
37	0.914	0.924	0.936	0.946	0.970
38	0.916	0.925	0.938	0.947	0.971
39	0.917	0.927	0.939	0.948	0.971
40	0.919	0.928	0.940	0.949	0.972
41	0.920	0.929	0.941	0.950	0.972
42	0.922	0.930	0.942	0.951	0.972
43	0.923	0.932	0.943	0.951	0.973
44	0.924	0.933	0.944	0.952	0.973
45	0.926	0.934	0.945	0.953	0.973
46	0.927	0.935	0.945	0.953	0.974
47	0.928	0.936	0.946	0.954	0.974
48	0.929	0.937	0.947	0.954	0.974
49	0.929	0.937	0.947	0.955	0.974
50	0.930	0.938	0.947	0.955	0.974

TABLE IX

Constants Used in Obtaining Probability of Calculated W in Test for Normality

n	γ	η	ϵ	n	γ	η	ϵ
3	−0.625	0.386	0.7500	27	−5.905	1.905	0.1980
4	−1.107	0.714	0.6297	28	−5.988	1.919	0.1943
5	−1.530	0.935	0.5521	29	−6.074	1.934	0.1907
6	−2.010	1.138	0.4963	30	−6.160	1.949	0.1872
7	−2.356	1.245	0.4533	31	−6.248	1.965	0.1840
8	−2.696	1.333	0.4186	32	−6.324	1.976	0.1811
9	−2.968	1.400	0.3900	33	−6.402	1.988	0.1781
10	−3.262	1.471	0.3660	34	−6.480	2.000	0.1755
11	−3.485	1.515	0.3451	35	−6.559	2.012	0.1727
12	−3.731	1.571	0.3270	36	−6.640	2.024	0.1702
13	−3.936	1.613	0.3111	37	−6.721	2.037	0.1677
14	−4.155	1.655	0.2969	38	−6.803	2.049	0.1656
15	−4.373	1.695	0.2842	39	−6.887	2.062	0.1633
16	−4.567	1.724	0.2727	40	−6.961	2.075	0.1612
17	−4.713	1.739	0.2622	41	−7.035	2.088	0.1591
18	−4.885	1.770	0.2528	42	−7.111	2.101	0.1572
19	−5.018	1.786	0.2440	43	−7.188	2.114	0.1552
20	−5.153	1.802	0.2359	44	−7.266	2.128	0.1534
21	−5.291	1.818	0.2264	45	−7.345	2.141	0.1516
22	−5.413	1.835	0.2207	46	−7.414	2.155	0.1499
23	−5.508	1.848	0.2157	47	−7.484	2.169	0.1482
24	−5.605	1.862	0.2106	48	−7.555	2.183	0.1466
25	−5.704	1.876	0.2063	49	−7.615	2.198	0.1451
26	−5.803	1.890	0.2020	50	−7.677	2.212	0.1436

APPENDIX D

COMPLETE RESULTS OF DATA ANALYSIS

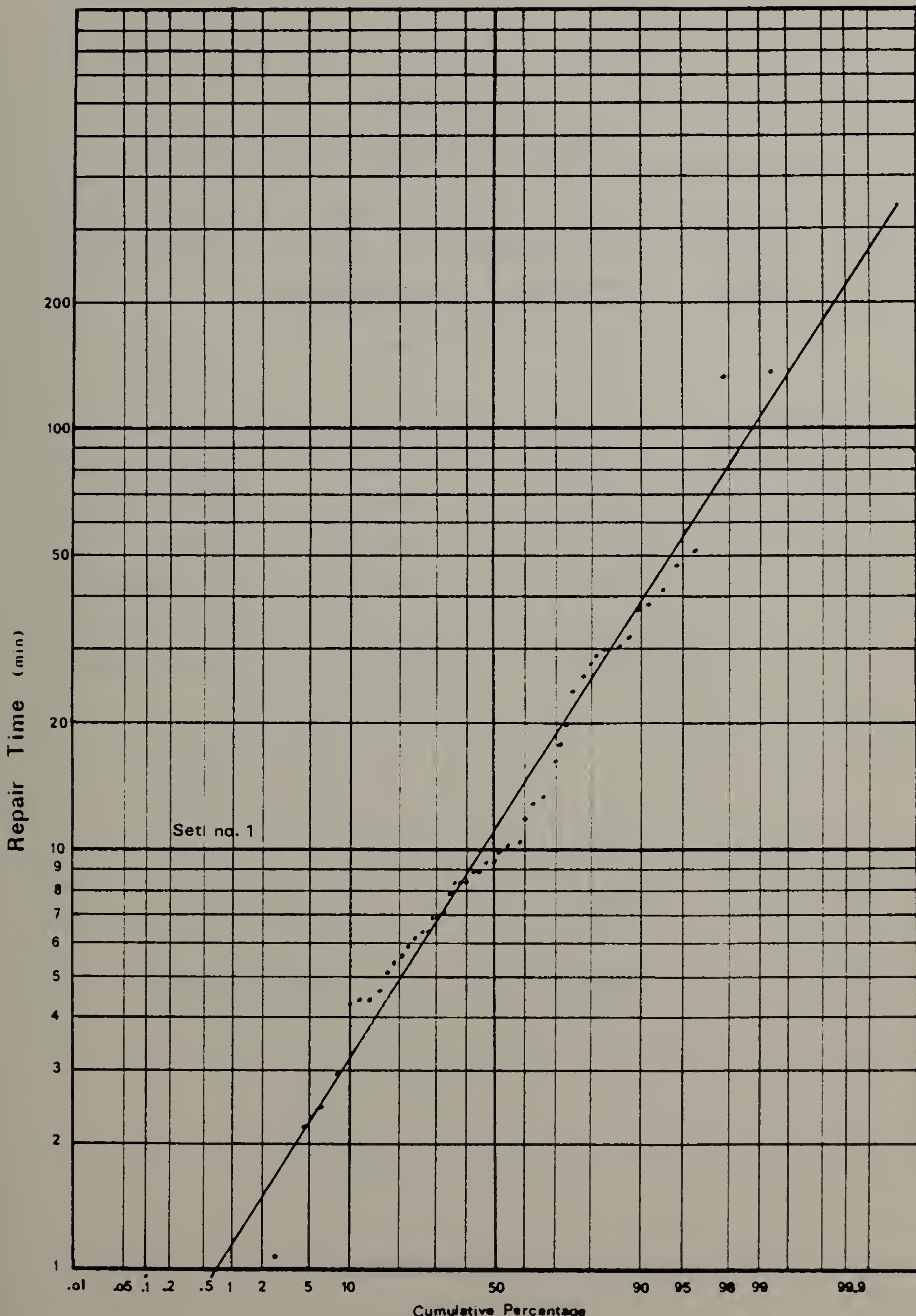
Set No. 1

AN/TRC-87 COMMUNICATIONS TRANCEIVER

SAMPLE SIZE N = 59   NO. OF CELLS K = 11

SAMPLE MEAN = 18.67   STANDARD DEV = 24.82

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.05	2.43	
PARAM2		0.95	
MTTR	18.67	18.30	2.04 %
50-TH PERCNT	12.94	11.37	13.82 %
90-TH PERCNT	42.98	39.70	8.27 %
95-TH PERCNT	55.92	56.57	1.15 %
CHI-SQR STAT	24.34	10.54	
DEG OF FREED	9	8	
SIGNIF LEVEL	0.380E-02	0.229E 00	


Repair Time (min)
Setl no. 1
Cumulative Percentage
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9

Set No. 2

QUICK REACTION CAPABILITY RADAR

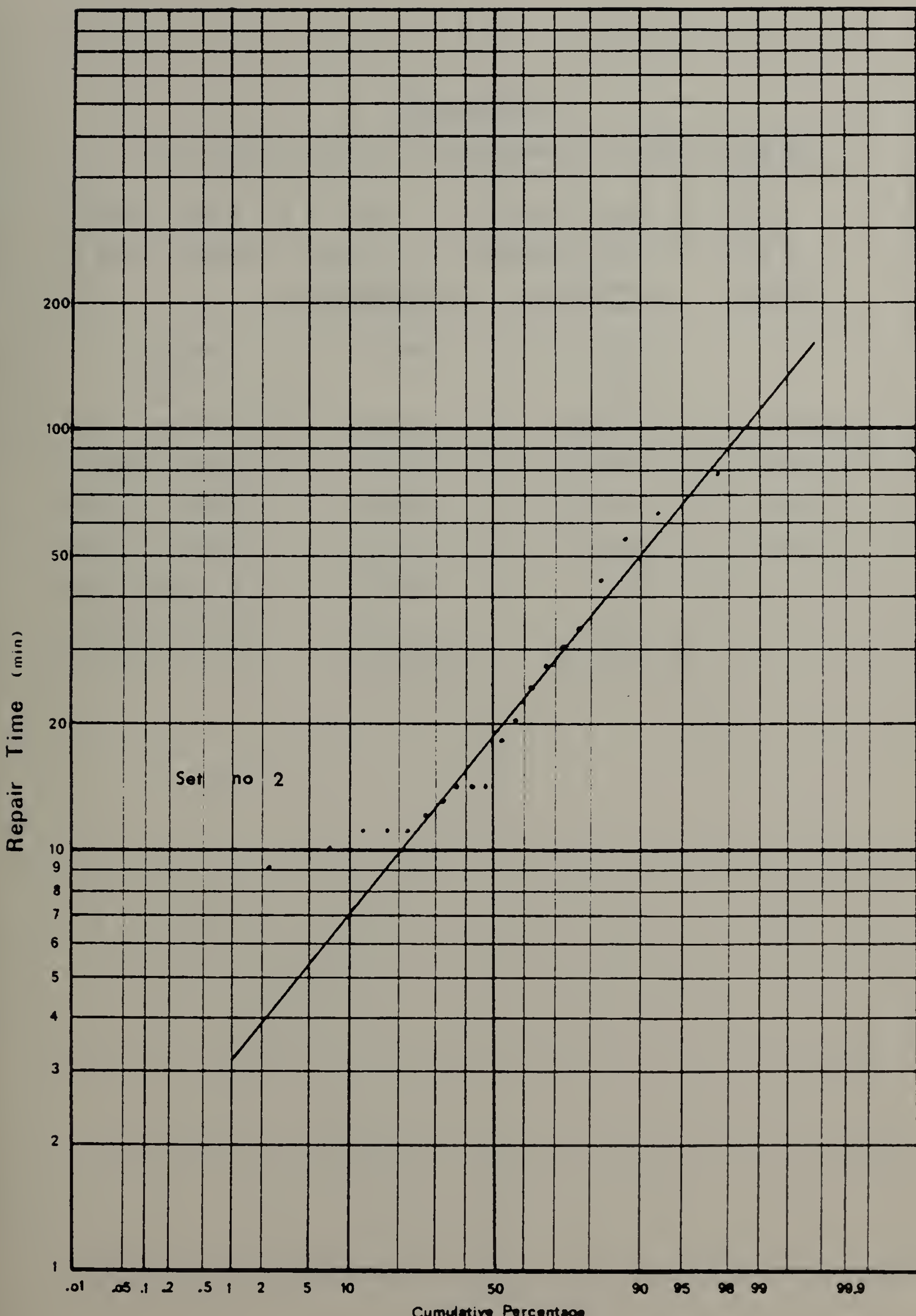
SAMPLE SIZE N = 20 NO. OF CELLS K = 4

SAMPLE MEAN = 25.35 STANDARD DEV = 19.40

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.04	3.01	
PARAM2		0.43	
MTTR	25.35	25.12	0.91 %
50-TH PERCNT	17.57	20.23	13.12 %
90-TH PERCNT	58.37	47.04	24.09 %
95-TH PERCNT	75.94	59.74	27.12 %
CHI-SQR STAT	10.40	1.20	
DEG OF FREED	2	1	
SIGNIF LEVEL	0.552E-02	0.273E 00	

W-TEST

b^2	7.47
S^2	8.23
W STAT	0.907
SIGNIF LEVEL	0.06



Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
Set no 2
.01 .05 .1 .2 .5 1 2 5 10 50 90 95 98 99 99.9
Cumulative Percentage

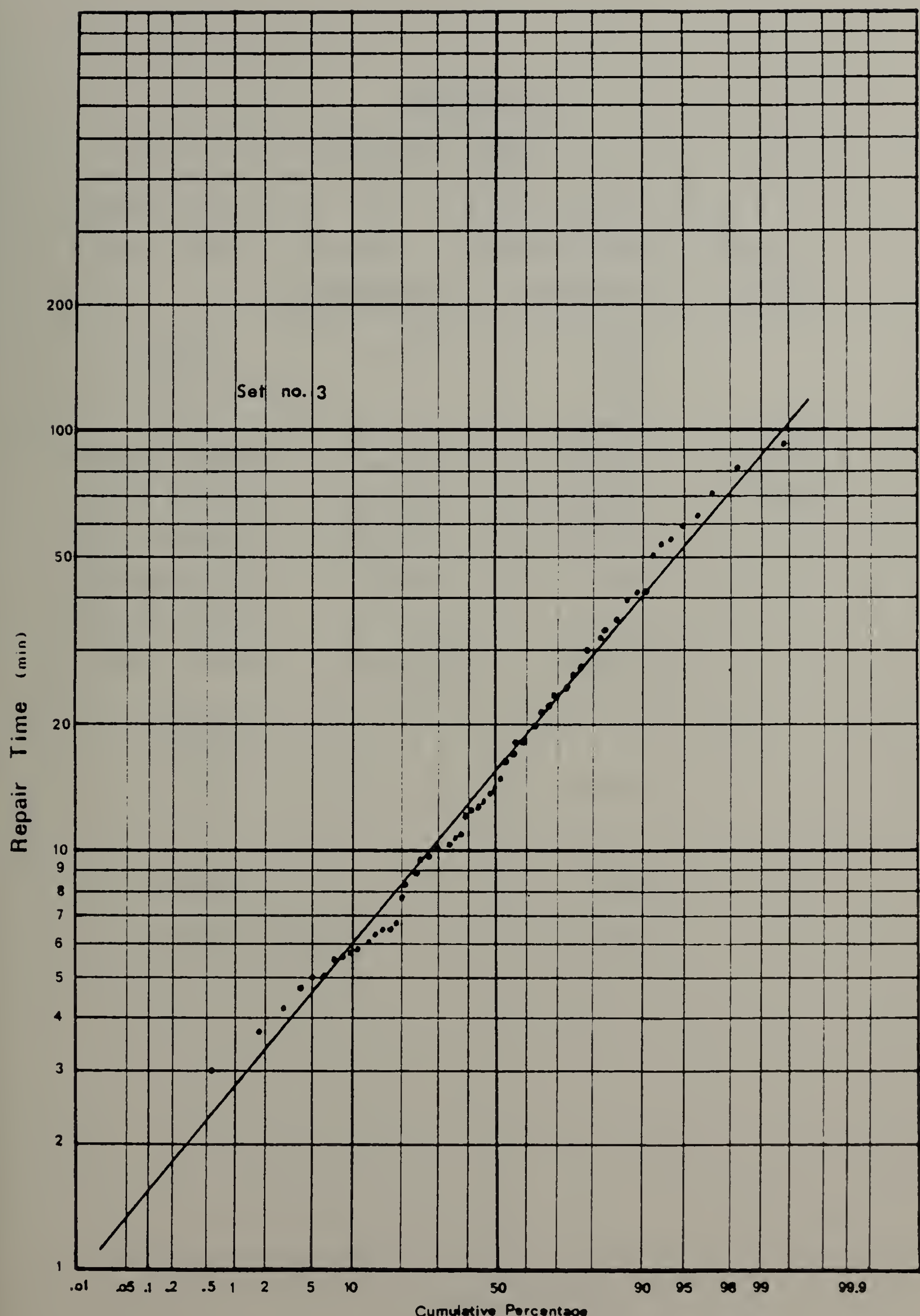
Set No. 3

AN/GSA-51 BACK UP INTERCEPTOR CONTROL SYSTEM

SAMPLE SIZE N = 90 NO. OF CELLS K = 18

SAMPLE MEAN = 20.43 STANDARD DEV = 17.07

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.05	2.73	
PARAM2		0.57	
MTTR	20.43	20.42	0.06 %
50-TH PERCNT	14.16	15.33	7.64 %
90-TH PERCNT	47.04	40.46	16.28 %
95-TH PERCNT	61.21	53.25	14.94 %
CHI-SQR STAT	25.60	7.60	
DEG OF FREED	16	15	
SIGNIF LEVEL	0.599E-01	0.939E 00	


Set no. 3
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 4

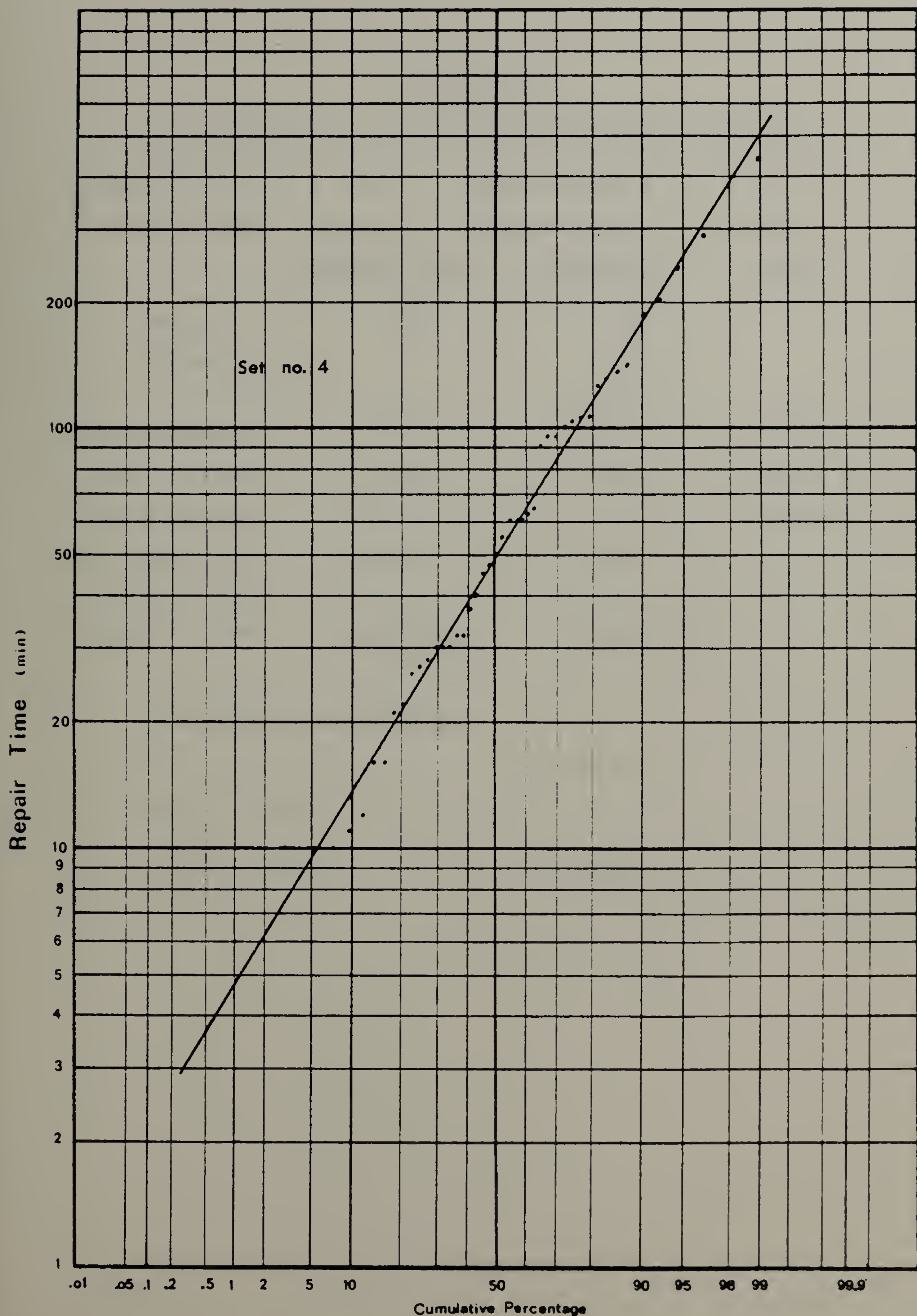
AN/FPS - 80 (TRACKING RADAR)

SAMPLE SIZE N = 45 NO. OF CELLS K = 9

SAMPLE MEAN = 78.13 STANDARD DEV = 83.49

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.01	3.89	
PARAM2		1.00	
MTTR	78.13	80.97	3.50 %
50-TH PERCNT	54.16	49.02	10.48 %
90-TH PERCNT	179.91	177.07	1.60 %
95-TH PERCNT	234.07	254.73	8.11 %
CHI-SQR STAT	10.80	2.00	
DEG OF FREED	7	6	
SIGNIF LEVEL	0.148E 00	0.920E 00	

W-TEST	
b^2	43.30
S^2	44.18
W STAT	0.979
SIGNIF LEVEL	0.71



Set no. 4
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 5

AN/TPS-39 (V) RADAR SURVEILLANCE SYSTEM

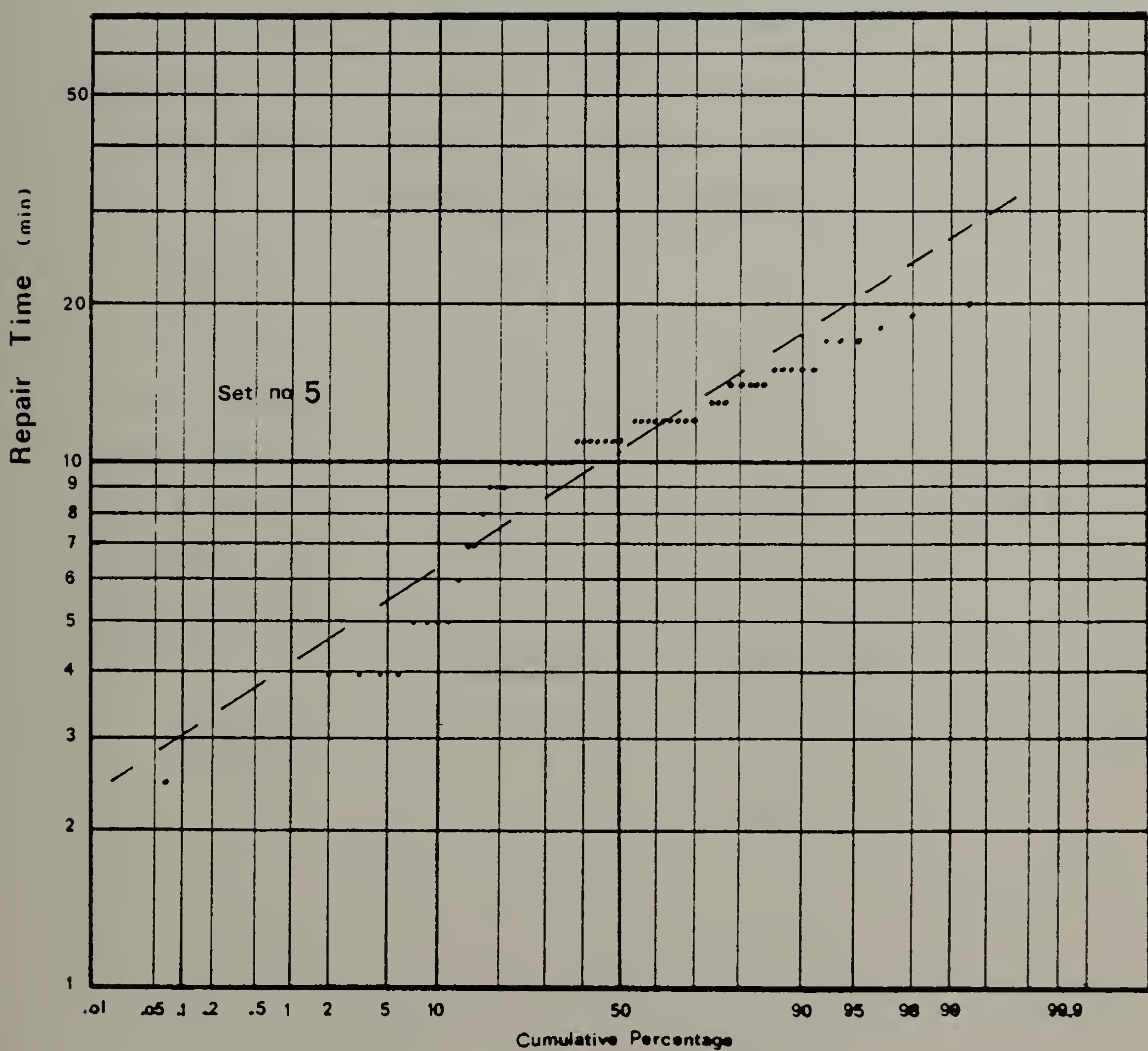
SAMPLE SIZE N = 75 NO. OF CELLS K = 15

SAMPLE MEAN = 11.11 STANDARD DEV = 3.62

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.09	2.34	
PARAM2		0.16	
MTTR	11.11	11.27	1.39 %
50-TH PERCNT	7.70	10.38	25.80 %
90-TH PERCNT	25.59	17.45	46.62 %
95-TH PERCNT	33.29	20.22	64.66 %
CHI-SQR STAT	148.40	62.80	
DEG OF FREED	13	12	
SIGNIF LEVEL	0.433E-24	0.694E-08	

χ^2 Test for Normality

χ^2 STAT	65.60
DEG OF FREED	12
SIGNIF LEVEL	< 0.005


Repair Time (min)
50
20
10
9
8
7
6
5
4
3
2
1
Set no 5
.01 .05 .1 .2 .5 1 2 5 10 50 90 95 98 99 99.9
Cumulative Percentage

Set No. 6

AM/3949-GR RADIO FREQUENCY AMPLIFIER

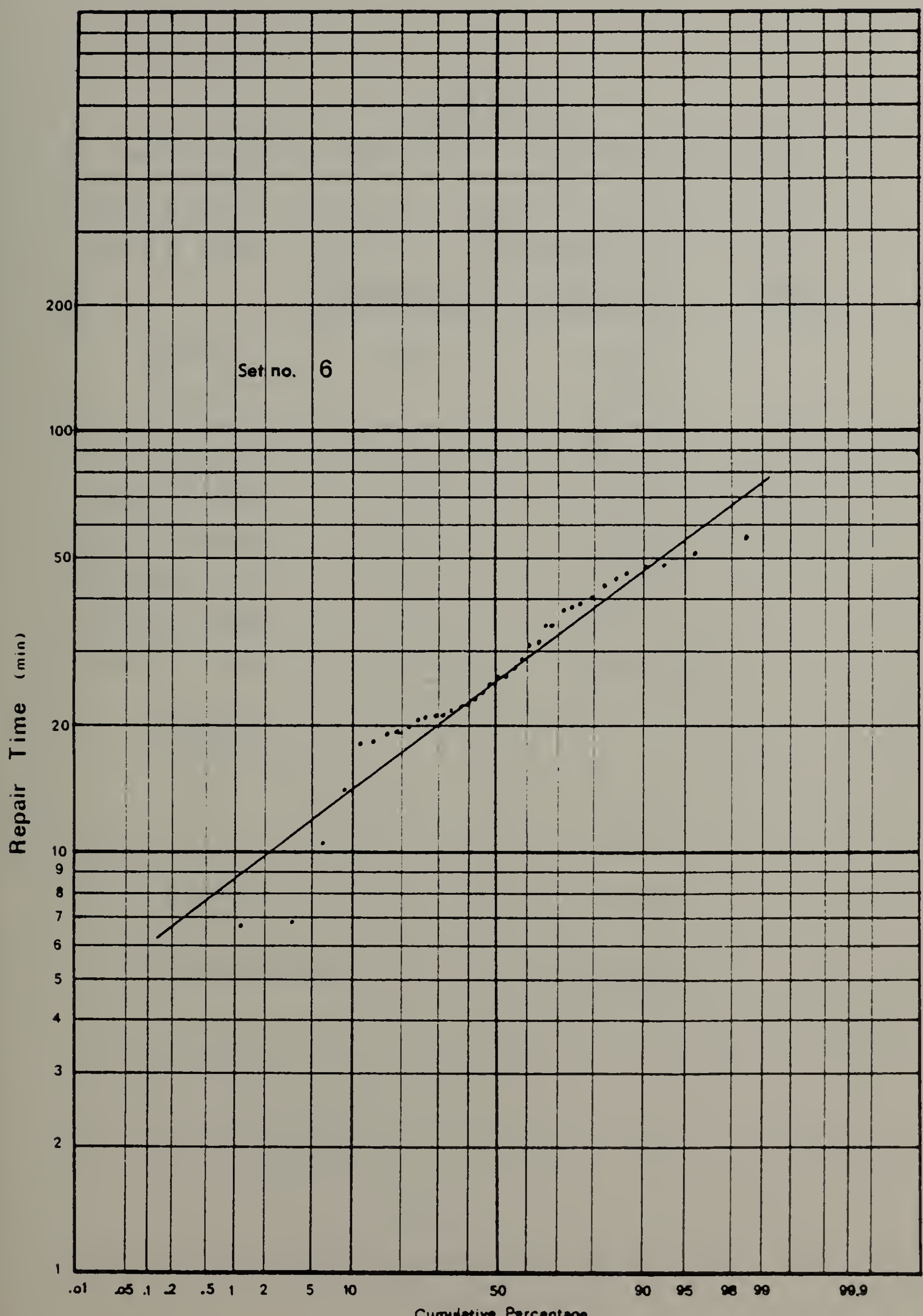
SAMPLE SIZE N = 38 NO. OF CELLS K = 7

SAMPLE MEAN = 28.49 STANDARD DEV = 12.30

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.04	3.24	
PARAM2		0.25	
MTTR	28.49	29.01	1.77 %
50-TH PERCNT	19.75	25.63	22.95 %
90-TH PERCNT	65.61	48.50	35.28 %
95-TH PERCNT	85.36	58.10	46.93 %
CHI-SQR STAT	33.11	4.74	
DEG OF FREED	5	4	
SIGNIF LEVEL	0.359E-05	0.315E 00	

W-TEST

b^2	8.63
S^2	9.15
W STAT	0.943
SIGNIF LEVEL	0.074


Set no. 6
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01 .05 .1 .2 .5 1 2 5 10 50 90 95 98 99 99.9
Cumulative Percentage

Set No. 7

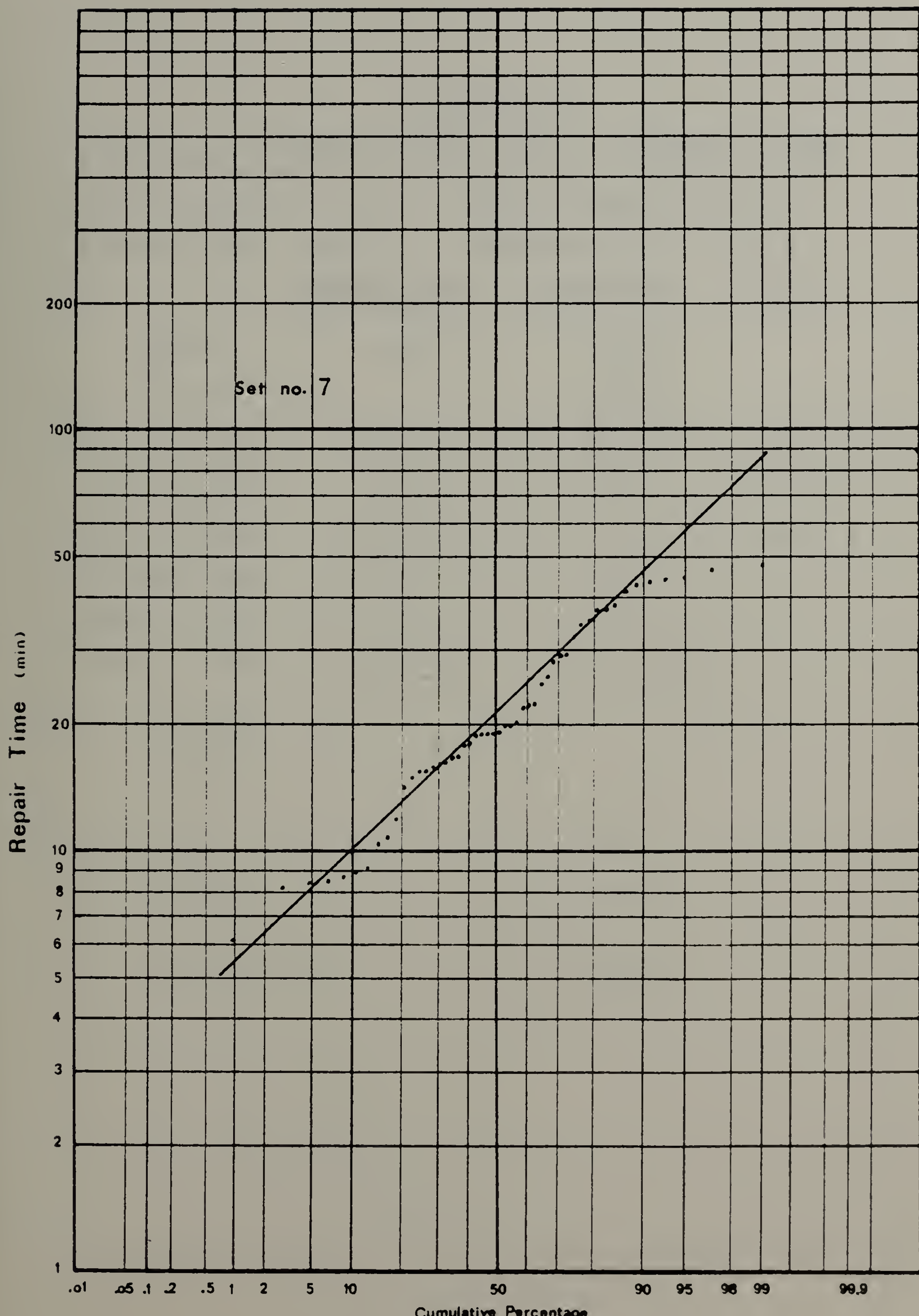
AN/ARC-164(V) RADIO SET - INTERMEDIATE LEVEL

SAMPLE SIZE N = 50 NO. OF CELLS K = 10

SAMPLE MEAN = 22.44 STANDARD DEV = 11.56

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.04	2.97	
PARAM2		0.29	
MTTR	22.44	22.65	0.94 %
50-TH PERCNT	15.55	19.59	20.61 %
90-TH PERCNT	51.66	39.09	32.17 %
95-TH PERCNT	67.21	47.53	41.40 %
CHI-SQR STAT	41.60	11.20	
DEG OF FREED	8	7	
SIGNIF LEVEL	0.161E-05	0.130E 00	

W-TEST	
b^2	13.49
s^2	14.21
W STAT	0.949
SIGNIF LEVEL	0.06


Set no. 7
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 8a

AN/ASN-131 AIRBORNE NAVIG. SYST. - ORGANIZ. LEVEL

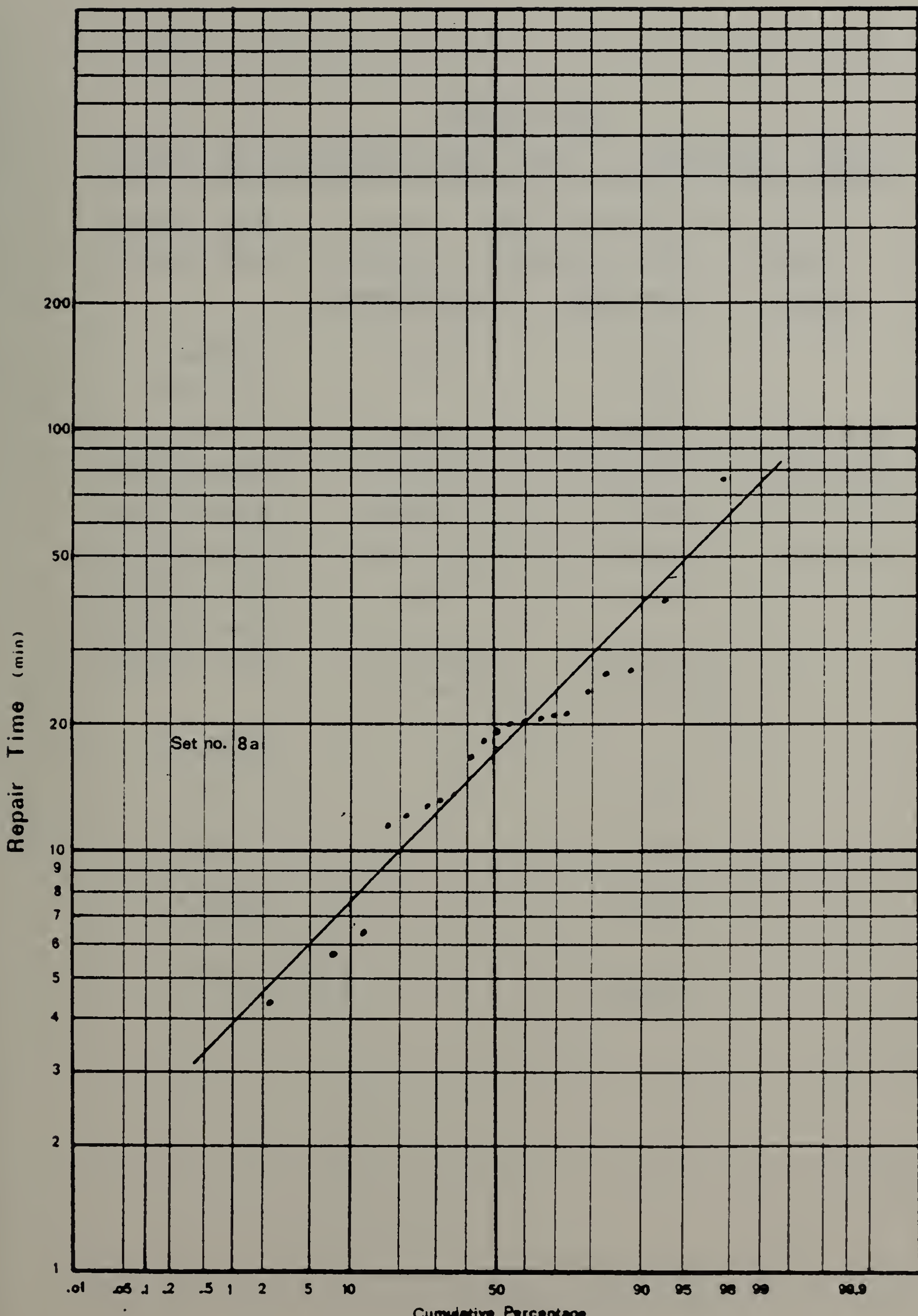
SAMPLE SIZE N = 21 NO. OF CELLS K = 4

SAMPLE MEAN = 20.21 STANDARD DEV = 14.78

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.05	2.32	
PARAM2		0.39	
MTTR	20.21	20.34	0.61 %
50-TH PERCNT	14.01	16.72	16.21 %
90-TH PERCNT	46.55	37.30	24.79 %
95-TH PERCNT	60.56	46.81	29.36 %
CHI-SQR STAT	10.43	2.31	
DEG OF FREED	2	1	
SIGNIF LEVEL	0.544E-02	0.937E-01	

W-TEST

b^2	7.40
s^2	7.84
W STAT	0.944
SIGNIF LEVEL	0.25



Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
Set no. 8a
.01 .05 .1 .2 .5 1 2 5 10 50 90 95 98 99 99.9
Cumulative Percentage

Set No. 8b

AN/ASN-131 AIRBORNE NAVIG. SYST. - INTERMED. LEVEL

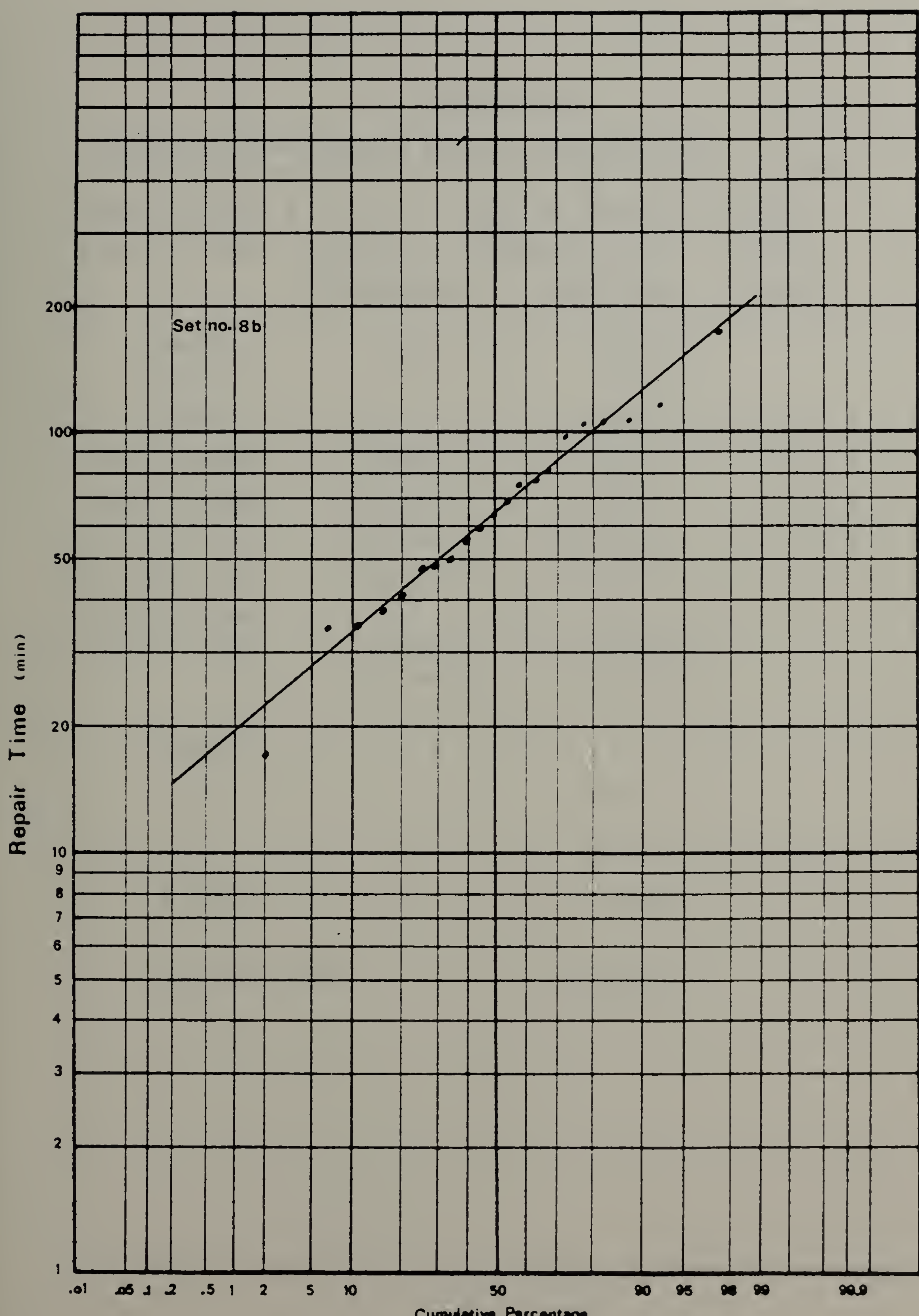
SAMPLE SIZE N = 21 NO. OF CELLS K = 4

SAMPLE MEAN = 70.67 STANDARD DEV = 36.06

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.01	4.13	
PARAM2		0.28	
MTTR	70.67	71.73	1.48 %
50-TH PERCNT	48.99	62.32	21.40 %
90-TH PERCNT	162.73	123.00	32.30 %
95-TH PERCNT	211.71	149.11	41.98 %
CHI-SQR STAT	6.24	0.14	
DEG OF FREED	2	1	
SIGNIF LEVEL	0.442E-01	0.705E 00	

W-TEST

b^2	5.50
S^2	5.62
W STAT	0.978
SIGNIF LEVEL	0.87


Set no. 8b
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 9

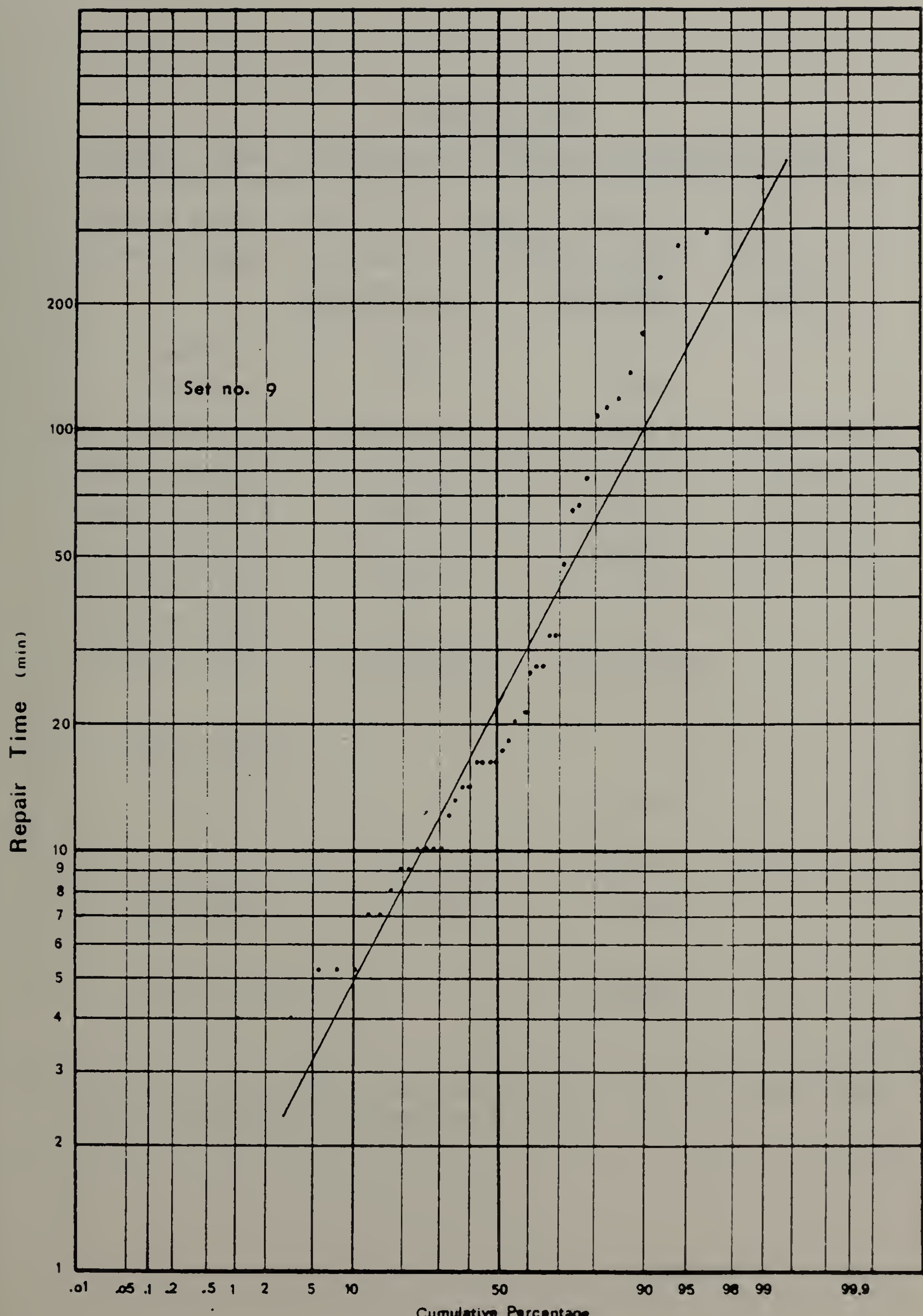
HARPOON SHIP COMMAND LAUNCH CONTROL SET

SAMPLE SIZE N = 44 NO. OF CELLS K = 8

SAMPLE MEAN = 56.25 STANDARD DEV = 85.65

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.02	3.19	
PARAM2		1.57	
MTTR	56.25	53.11	5.90 %
50-TH PERCNT	38.99	24.26	60.71 %
90-TH PERCNT	129.52	120.76	7.26 %
95-TH PERCNT	168.51	190.22	11.42 %
CHI-SQR STAT	23.27	10.91	
DEG OF FREED	6	5	
SIGNIF LEVEL	0.710E-03	0.532E-01	

W-TEST	
b^2	62.2
S^2	68.8
W STAT	0.904
SIGNIF LEVEL	< 0.01


Set no. 9
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 10a

DEFENCE COMM. SYSTEM/SCF INTERFACE SYSTEM-ON LINE

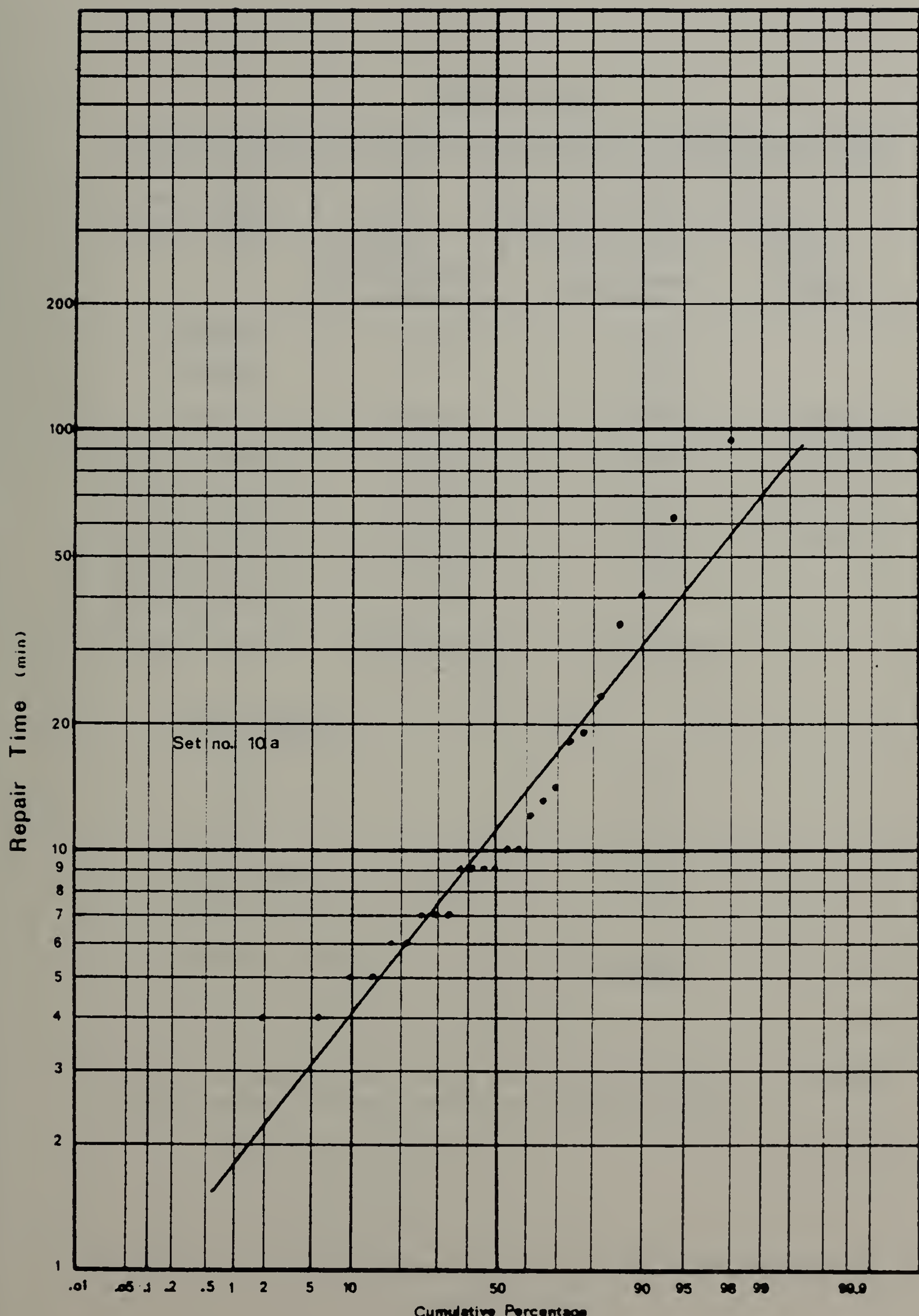
SAMPLE SIZE N = 25   NO. OF CELLS K = 5

SAMPLE MEAN = 17.40   STANDARD DEV = 20.60

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.06	2.45	
PARAM2		0.68	
MTTR	17.40	16.38	6.24 %
50-TH PERCNT	12.06	11.63	3.66 %
90-TH PERCNT	40.06	33.59	19.29 %
95-TH PERCNT	52.13	45.35	14.95 %
CHI-SQR STAT	12.40	4.40	
DEG OF FREED	3	2	
SIGNIF LEVEL	0.613E-02	0.111E 00	

W/WE TESTS		
b^2		15.05
s^2		16.42
W/WE STAT	0.018(*)	0.916
SIGNIF LEVEL		0.05

(*) A value lower than the "Lower Point" in the 95% Range which indicates non-exponentiality



Repair Time (min)
Set no. 10a
Cumulative Percentage
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9

Set No. 10b

DEFENCE COMM. SYSTEM/SCF INTERFACE SYSTEM-OFF LINE

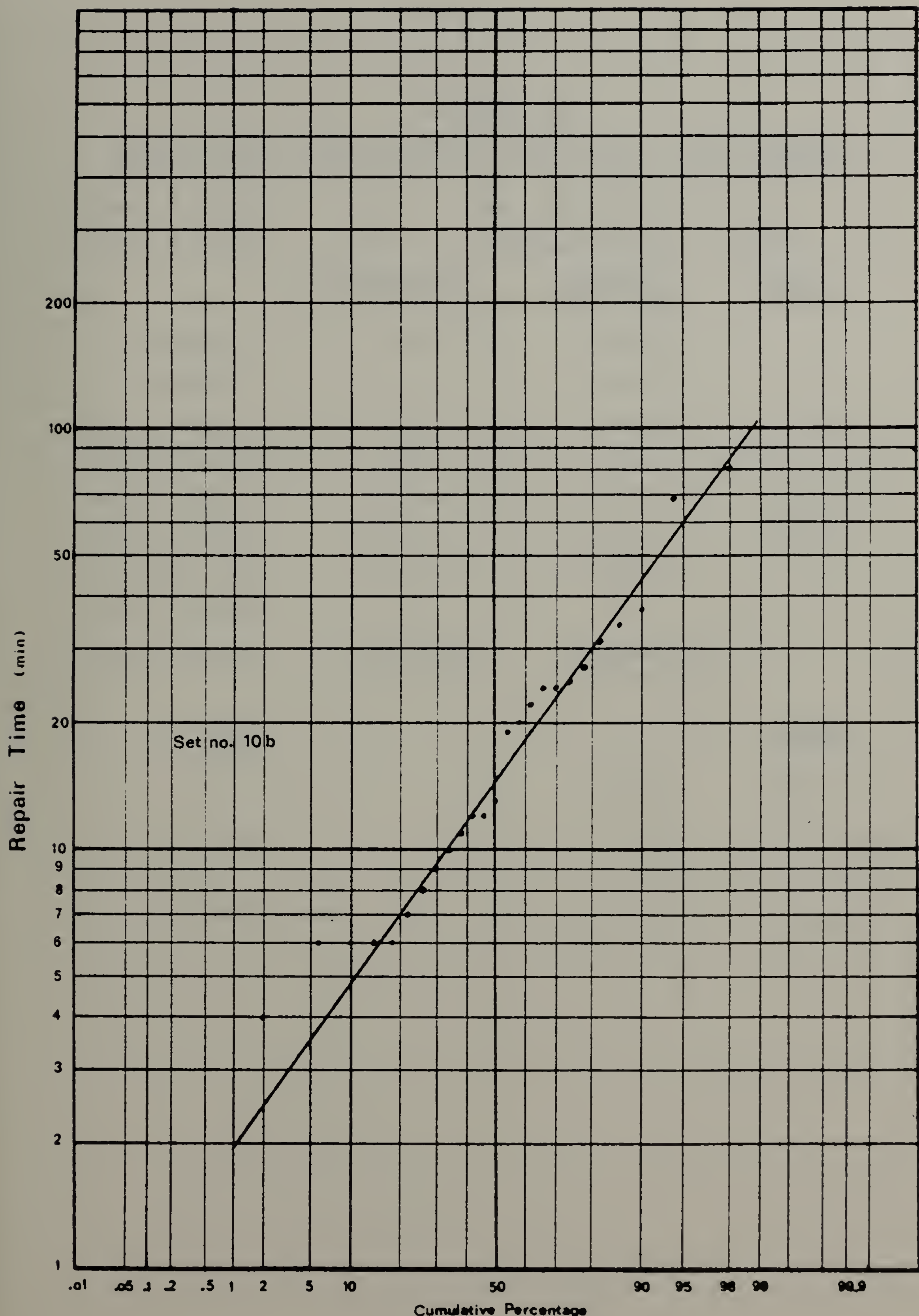
SAMPLE SIZE N = 25 NO. OF CELLS K = 5

SAMPLE MEAN = 20.84 STANDARD DEV = 18.75

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.05	2.72	
PARAM2		0.63	
MTTR	20.84	20.89	0.22 %
50-TH PERCNT	14.45	15.23	5.17 %
90-TH PERCNT	47.99	42.18	13.76 %
95-TH PERCNT	62.43	56.28	10.92 %
CHI-SQR STAT	6.00	4.40	
DEG OF FREED	3	2	
SIGNIF LEVEL	0.112E 00	0.111E 00	

W/WE TESTS		
b^2		14.54
s^2		15.14
W/WE STAT	0.034(*)	0.96
SIGNIF LEVEL		0.42

(*) A value between the Lower and Upper Point of the 90% or 95% Range, which indicates exponentiality.


Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
Set no. 10b
.01 .05 .1 .2 .5 1 2 5 10 50 90 95 98 99 99.9
Cumulative Percentage

Set No. 11

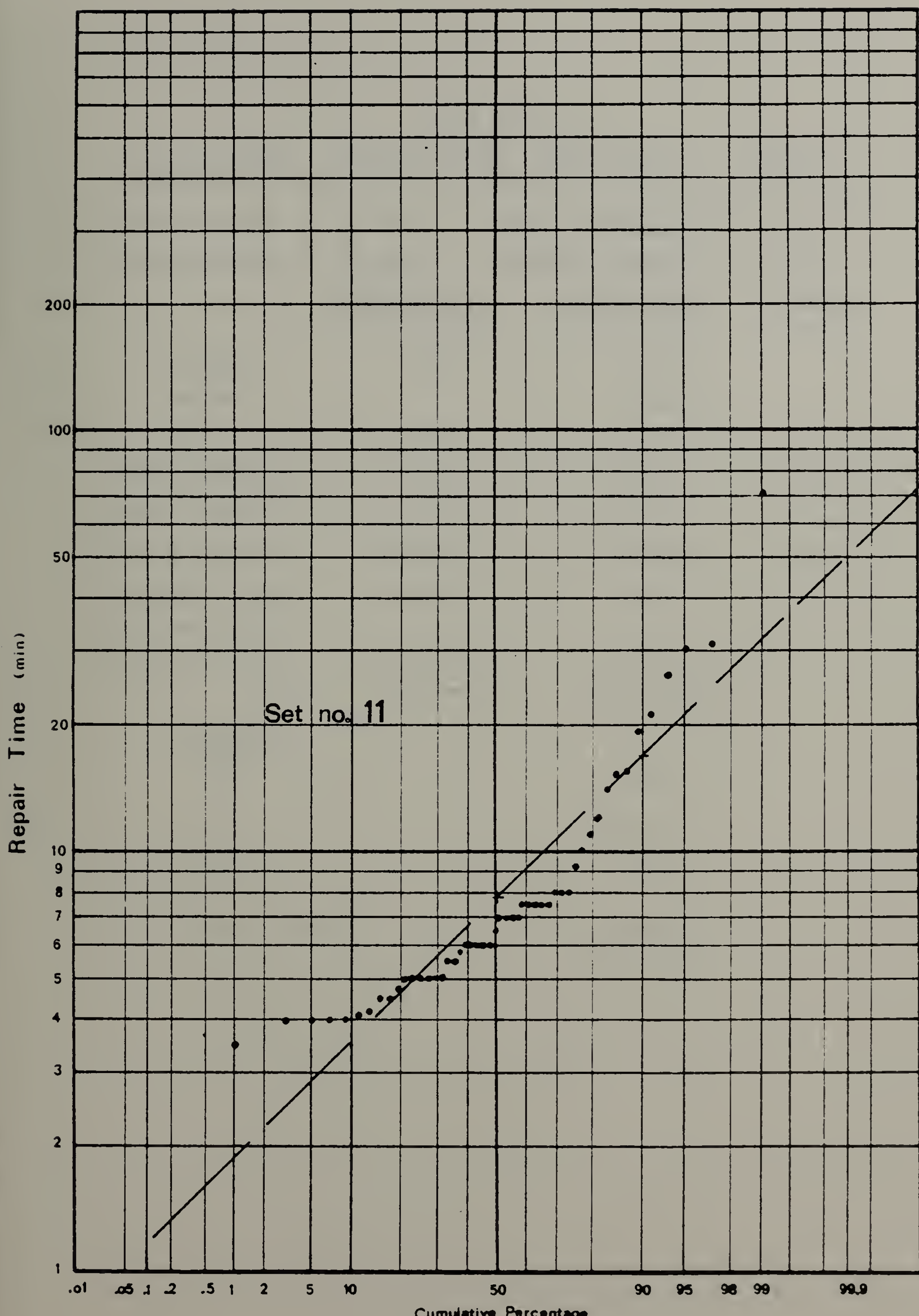
USASATCOMA COMMUNICATION SUBSYSTEM (CONTINGENCY)

SAMPLE SIZE N = 50 NO. OF CELLS K = 10

SAMPLE MEAN = 9.97 STANDARD DEV = 10.79

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.10	2.04	
PARAM2		0.40	
MTTR	9.97	9.34	6.71 %
50-TH PERCNT	6.91	7.66	9.79 %
90-TH PERCNT	22.96	17.18	33.60 %
95-TH PERCNT	29.87	21.60	38.28 %
CHI-SQR STAT	49.20	20.80	
DEG OF FREED	8	7	
SIGNIF LEVEL	0.582E-07	0.408E-02	

W TEST		NORMAL
b^2	16.72	2994.97
S^2	19.45	5704.80
W STAT	0.859	0.525
SIGNIF LEVEL	< 0.01	< 0.01


Set no. 11
Repair Time (min)
Cumulative Percentage
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9

Set No. 12

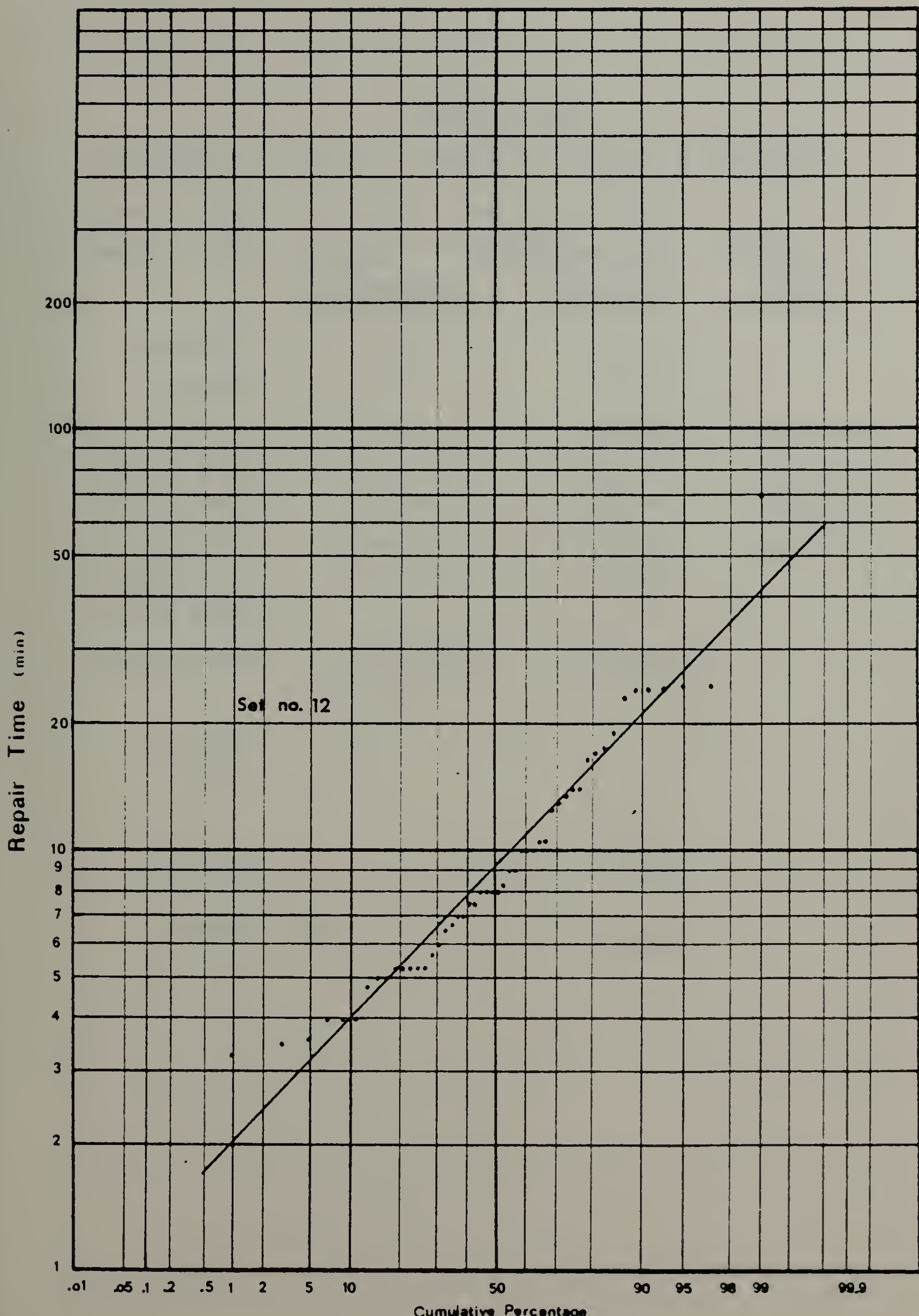
USASATCOMA COMMUNICATION SUBSYSTEM (NODAL)

SAMPLE SIZE N = 50 NO. OF CELLS K = 10

SAMPLE MEAN = 11.54 STANDARD DEV = 10.46

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.09	2.21	
PARAM2		0.42	
MTTR	11.54	11.22	2.81 %
50-TH PERCNT	8.00	9.10	12.08 %
90-TH PERCNT	26.57	20.88	27.23 %
95-TH PERCNT	34.56	26.42	30.82 %
CHI-SQR STAT	28.80	7.20	
DEG OF FREED	8	7	
SIGNIF LEVEL	0.344E-03	0.408E 00	

W-TEST	
b^2	19.54
S^2	20.58
W STAT	0.95
SIGNIF LEVEL	0.06


Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
Set no. 12
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 13a

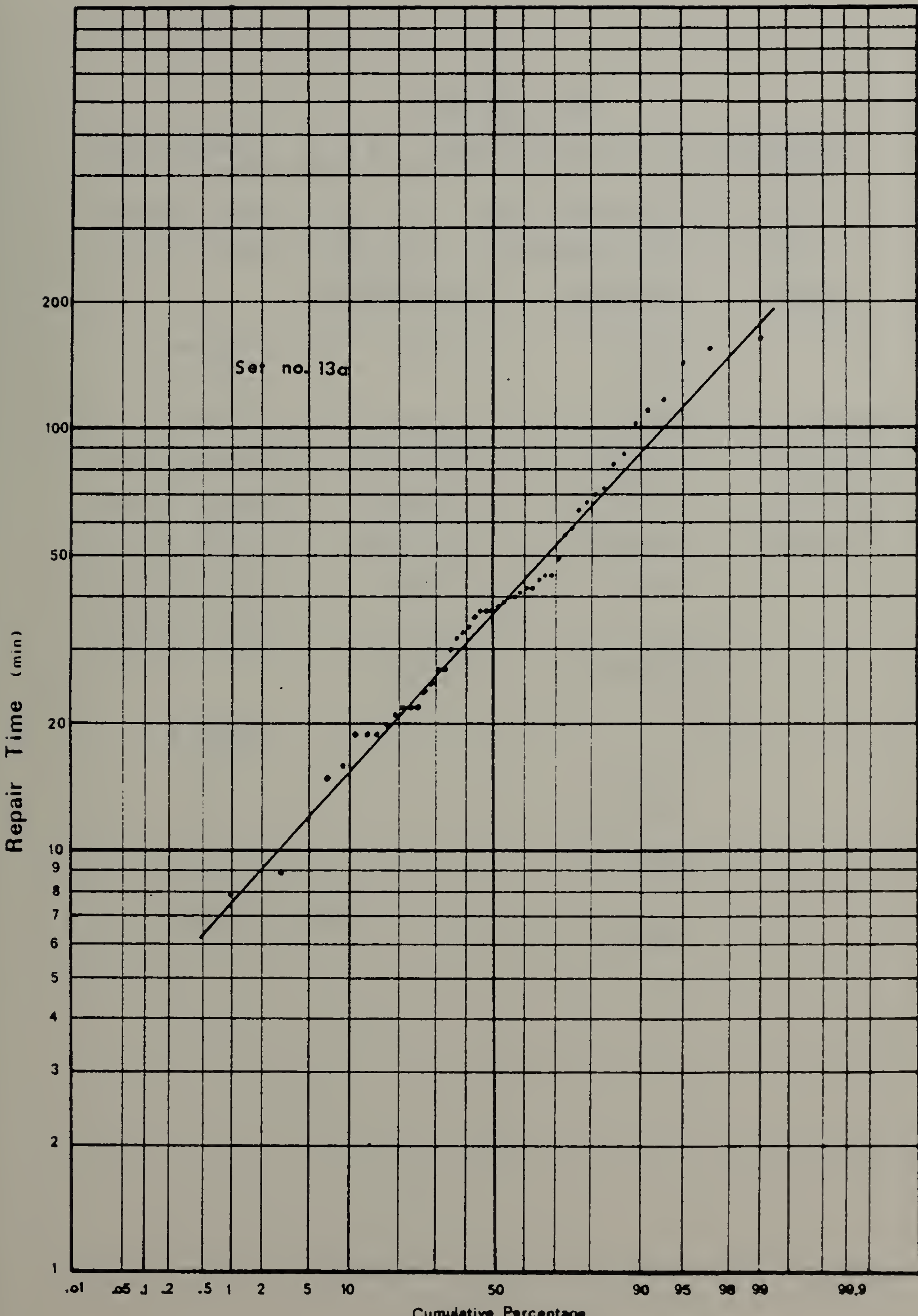
CONTINENTAL A D C - GROUND DATA SYS. (INHERENT)

SAMPLE SIZE N = 50 NO. OF CELLS K = 10

SAMPLE MEAN = 48.18 STANDARD DEV = 36.52

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.02	3.64	
PARAM2		0.48	
MTTR	48.18	48.30	0.24 %
50-TH PERCNT	33.40	37.97	12.05 %
90-TH PERCNT	110.94	92.39	20.08 %
95-TH PERCNT	144.33	118.84	21.45 %
CHI-SQR STAT	24.40	10.80	
DEG OF FREED	8	7	
SIGNIF LEVEL	0.196E-02	0.148E 00	

W-TEST	
b^2	23.01
S^2	23.57
W STAT	0.976
SIGNIF LEVEL	0.58


Set no. 13a
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 13b

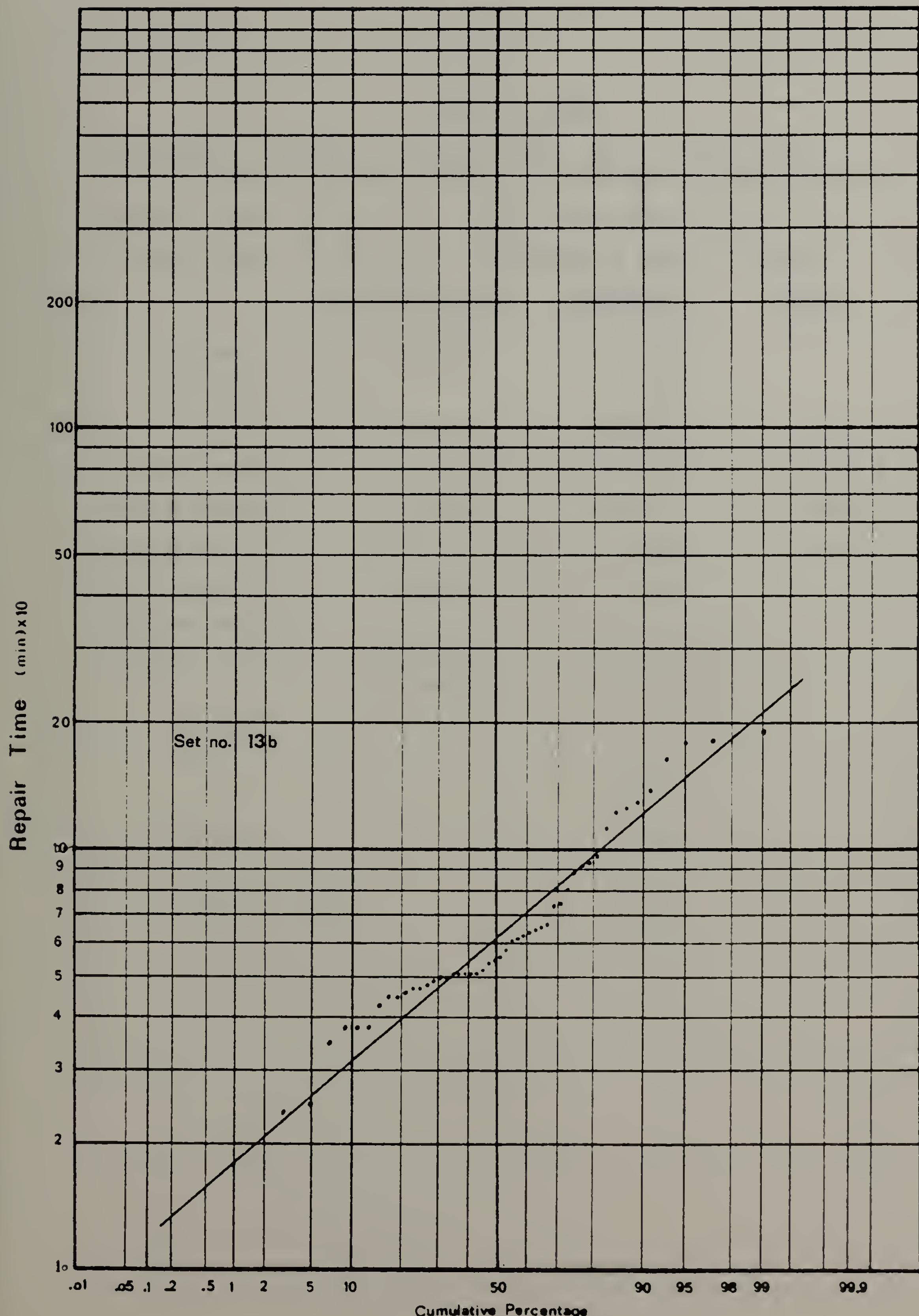
CONTINENTAL A D C - GROUND DATA SYS. (ACHIEVED)

SAMPLE SIZE N = 50 NO. OF CELLS K = 10

SAMPLE MEAN = 72.32 STANDARD DEV = 42.00

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.01	4.14	
PARAM2		0.27	
MTTR	72.32	72.07	0.35 %
50-TH PERCNT	50.13	62.97	20.39 %
90-TH PERCNT	166.52	122.58	35.85 %
95-TH PERCNT	216.65	148.02	46.36 %
CHI-SQR STAT	52.00	17.20	
DEG OF FREED	8	7	
SIGNIF LEVEL	0.168E-07	0.162E-01	

W-TEST	
b^2	12.57
S^2	13.23
W STAT	0.9505
SIGNIF LEVEL	0.067



Repair Time (min)x10
200
100
50
20
10
9
8
7
6
5
4
3
2
1
Set no. 13b
.01 .05 .1 .2 .5 1 2 5 10 50 90 95 98 99 99.9
Cumulative Percentage

Set No. 14a

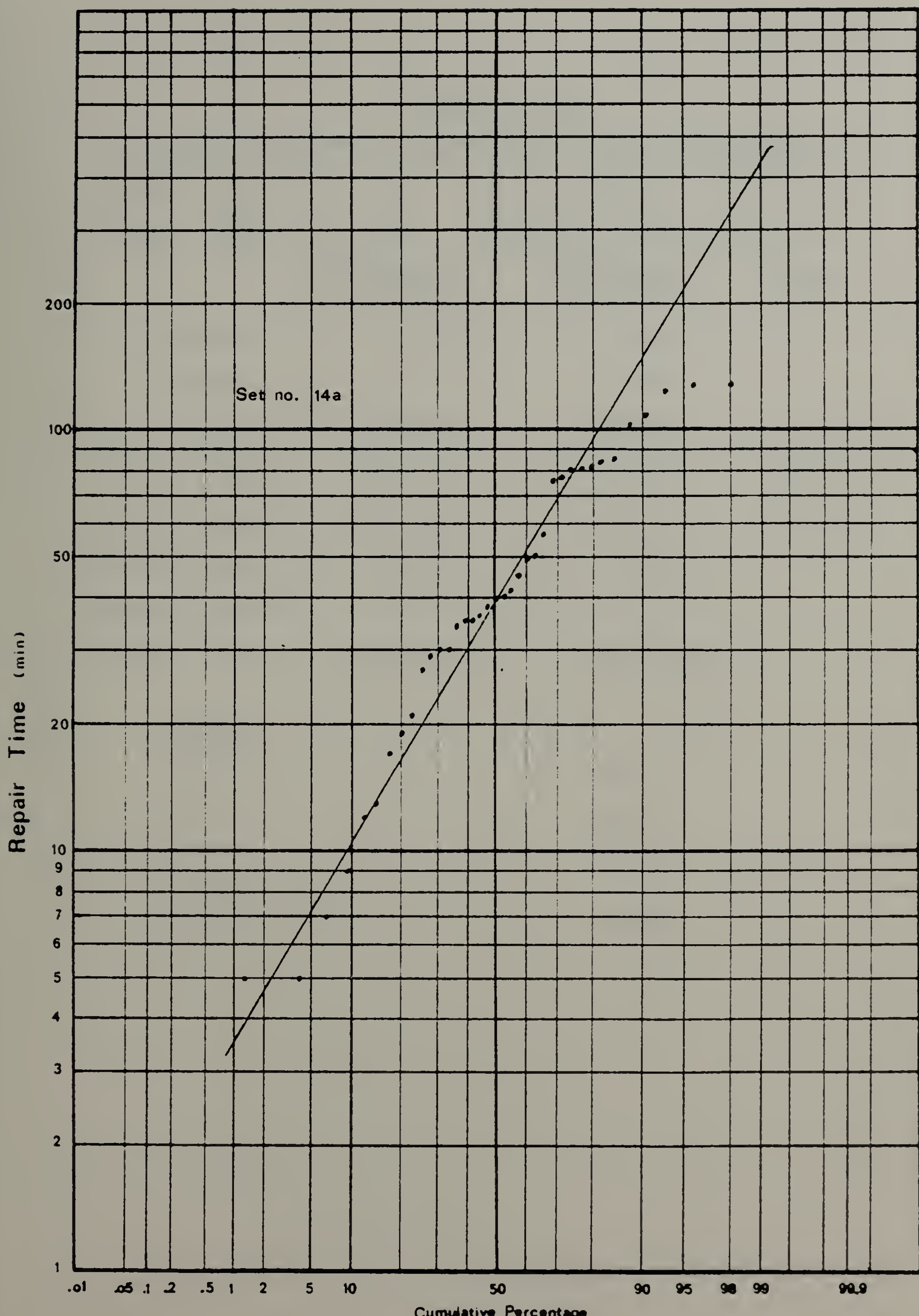
STRATEGIC AIRCOMM. GROUND DATA SYS. (INHERENT)

SAMPLE SIZE N = 37 NO. OF CELLS K = 7

SAMPLE MEAN = 50.41 STANDARD DEV = 35.54

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.02	3.61	
PARAM2		0.78	
MTTR	50.41	54.68	7.81 %
50-TH PERCNT	34.94	37.11	5.85 %
90-TH PERCNT	116.06	114.73	1.16 %
95-TH PERCNT	151.00	157.94	4.39 %
CHI-SQR STAT	9.35	3.68	
DEG OF FREED	5	4	
SIGNIF LEVEL	0.958E-01	0.452E 00	

W-TEST	
b^2	25.95
s^2	27.9
W STAT	0.93
SIGNIF LEVEL	0.03


Set no. 14a
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 14b

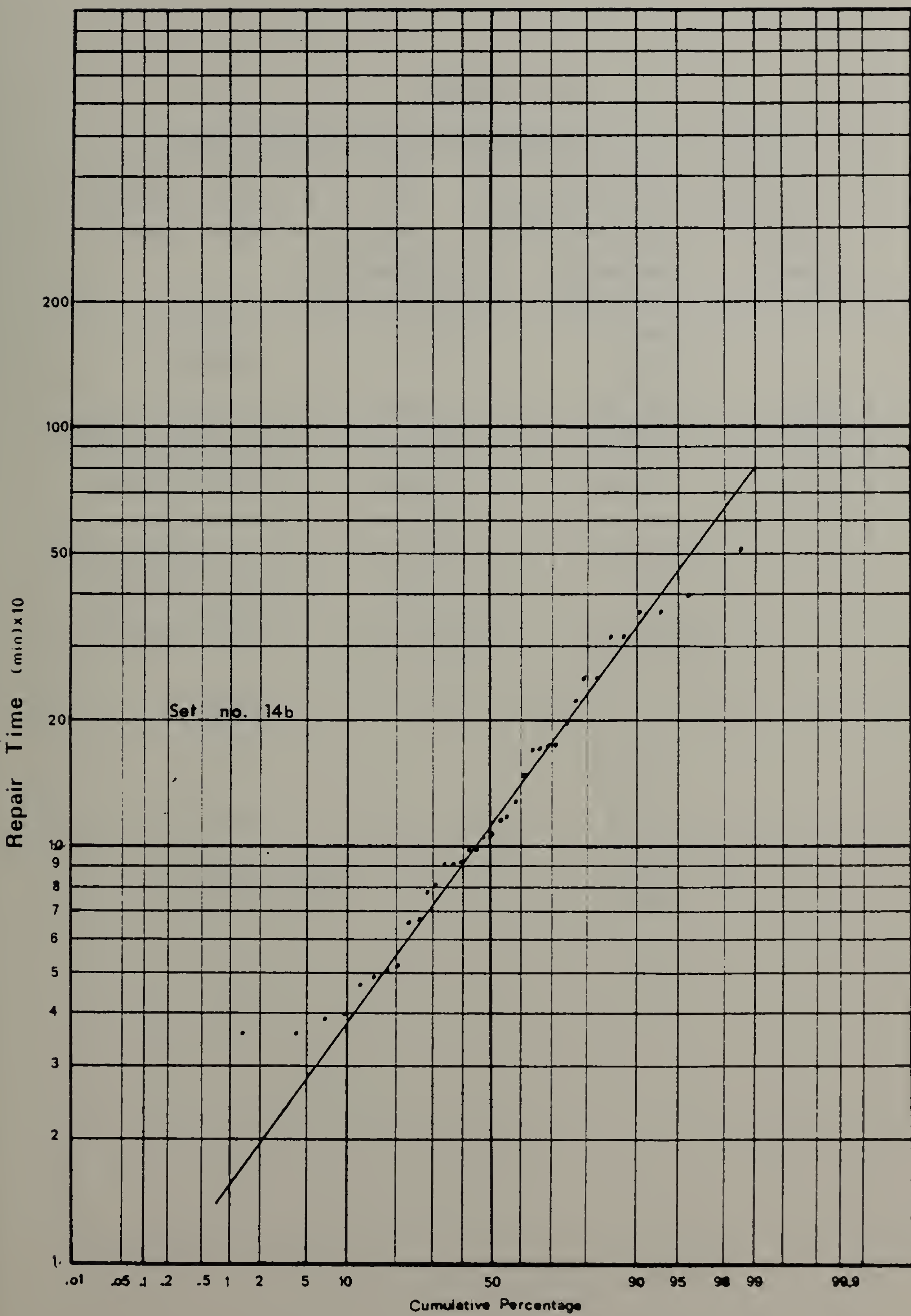
STRATEGIC AIRCOMM. GROUND DATA SYS. (ACHIEVED)

SAMPLE SIZE N = 37 NO. OF CELLS K = 7

SAMPLE MEAN = 154.00 STANDARD DEV = 117.22

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.01	4.77	
PARAM2		0.55	
MTTR	154.00	155.69	1.09 %
50-TH PERCNT	106.74	118.10	9.62 %
90-TH PERCNT	354.60	306.30	15.77 %
95-TH PERCNT	461.34	401.18	15.00 %
CHI-SQR STAT	10.49	4.43	
DEG OF FREED	5	4	
SIGNIF LEVEL	0.626E-01	0.351E 00	

W-TEST	
b^2	19.11
S^2	19.91
W STAT	0.96
SIGNIF LEVEL	0.26



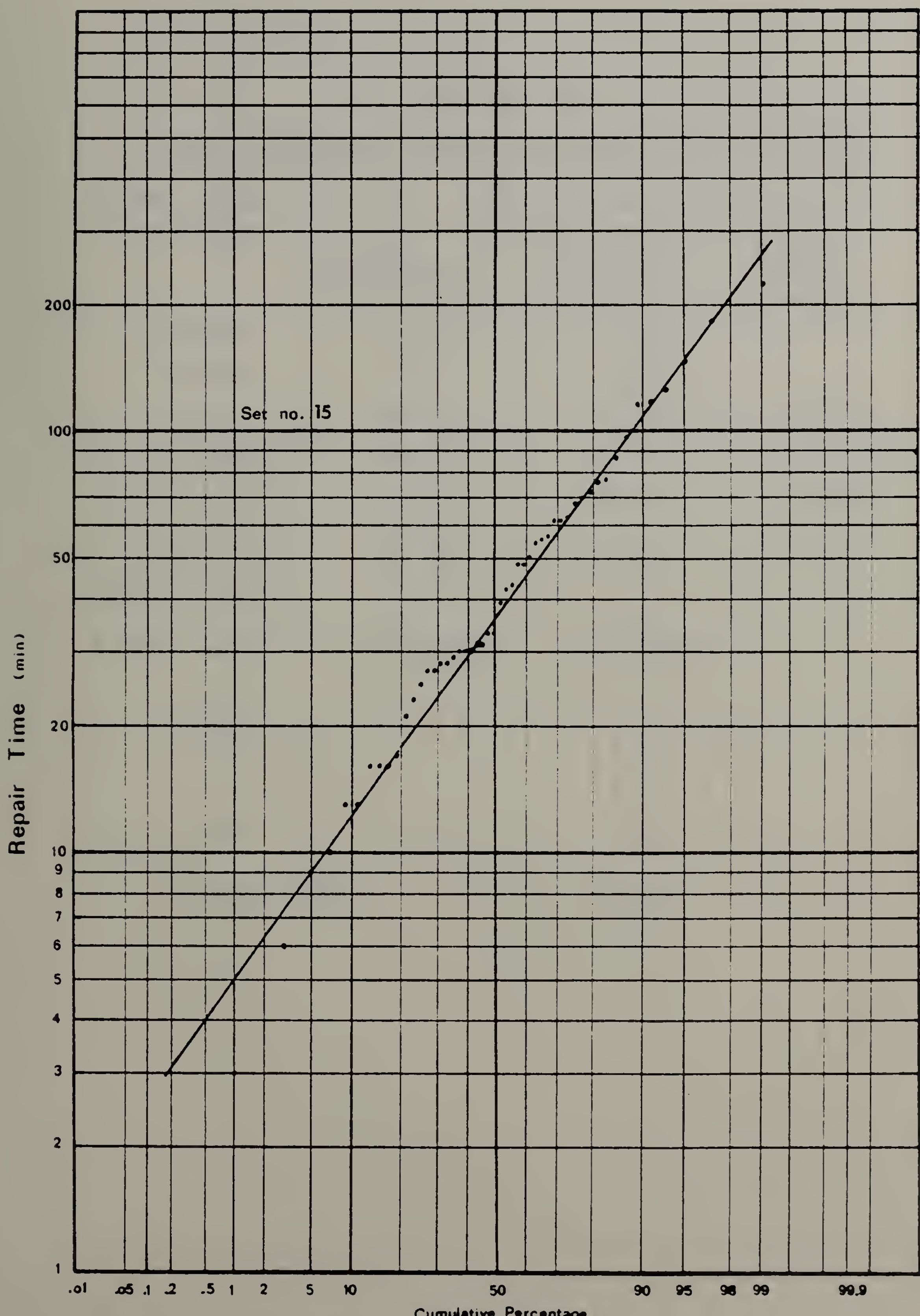
Repair Time (min)x10
Set no. 14b
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 15

SAMSO 46 FOOT TT&C ANTENNA

SAMPLE SIZE N = 50		NO. OF CELLS K = 10	
SAMPLE MEAN = 52.04		STANDARD DEV = 44.44	
	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.02	3.62	
PARAM2		0.74	
MTTR	52.04	54.34	4.24 %
50-TH PERCNT	36.07	37.47	3.74 %
90-TH PERCNT	119.83	113.18	5.87 %
95-TH PERCNT	155.90	154.77	0.73 %
CHI-SQR STAT	19.20	10.40	
DEG OF FREED	8	7	
SIGNIF LEVEL	0.138E-01	0.167E 00	

W-TEST	
b^2	35.82
S^2	36.41
W STAT	0.984
SIGNIF LEVEL	0.855


Set no. 15
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 16

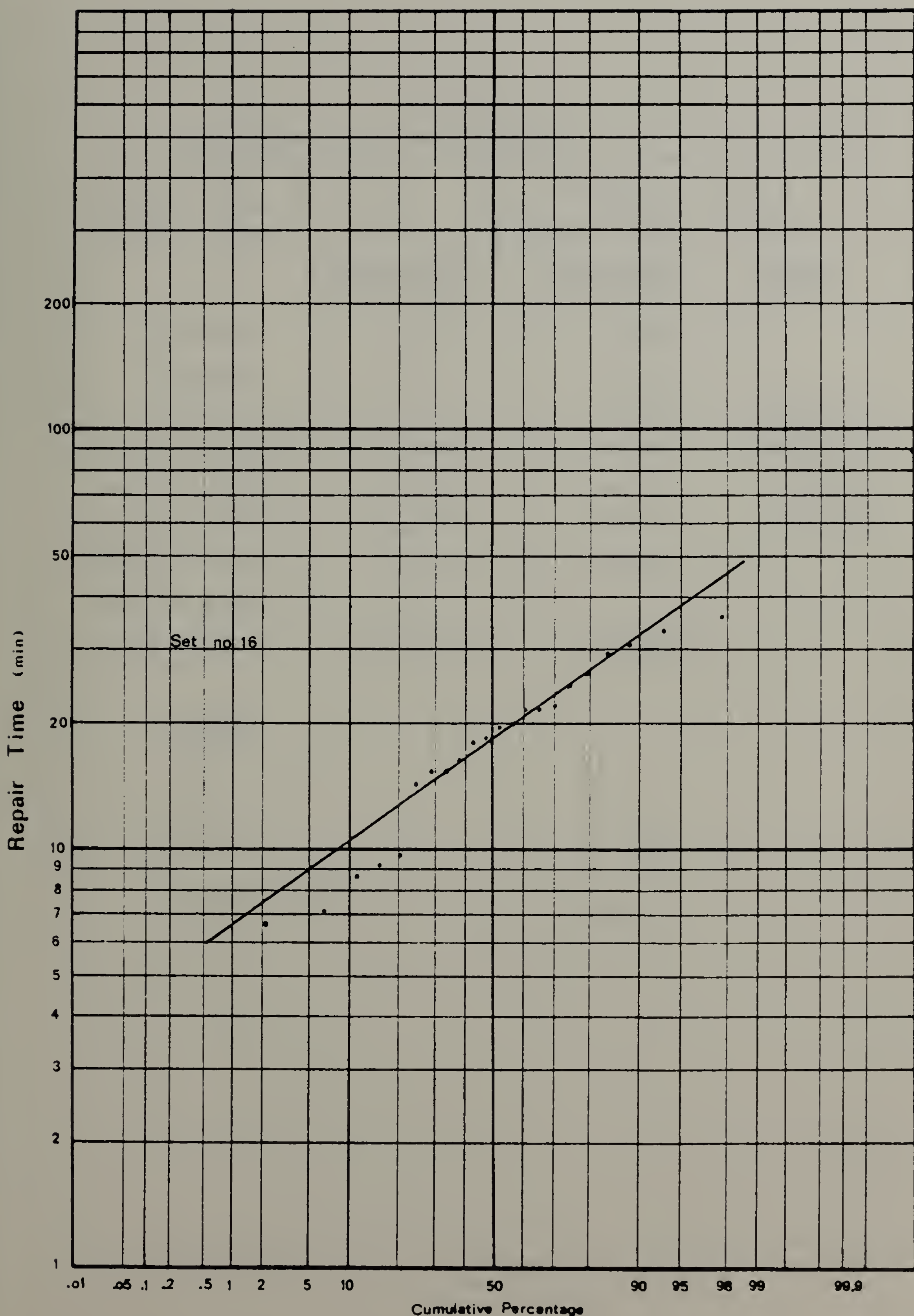
NADC DIGITAL TELEVISION PROJECTION UNIT

SAMPLE SIZE N = 22 NO. OF CELLS K = 4

SAMPLE MEAN = 19.02 STANDARD DEV = 8.73

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.05	2.83	
PARAM2		0.25	
MTTR	19.02	19.31	1.53 %
50-TH PERCNT	13.18	17.02	22.54 %
90-TH PERCNT	43.79	32.44	35.00 %
95-TH PERCNT	56.97	38.94	46.31 %
CHI-SQR STAT	16.18	0.91	
DEG OF FREED	2	1	
SIGNIF LEVEL	0.306E-03	0.340E 00	

W-TEST	
b^2	5.09
s^2	5.31
W STAT	0.957
SIGNIF LEVEL	0.43



Set no 16
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 17

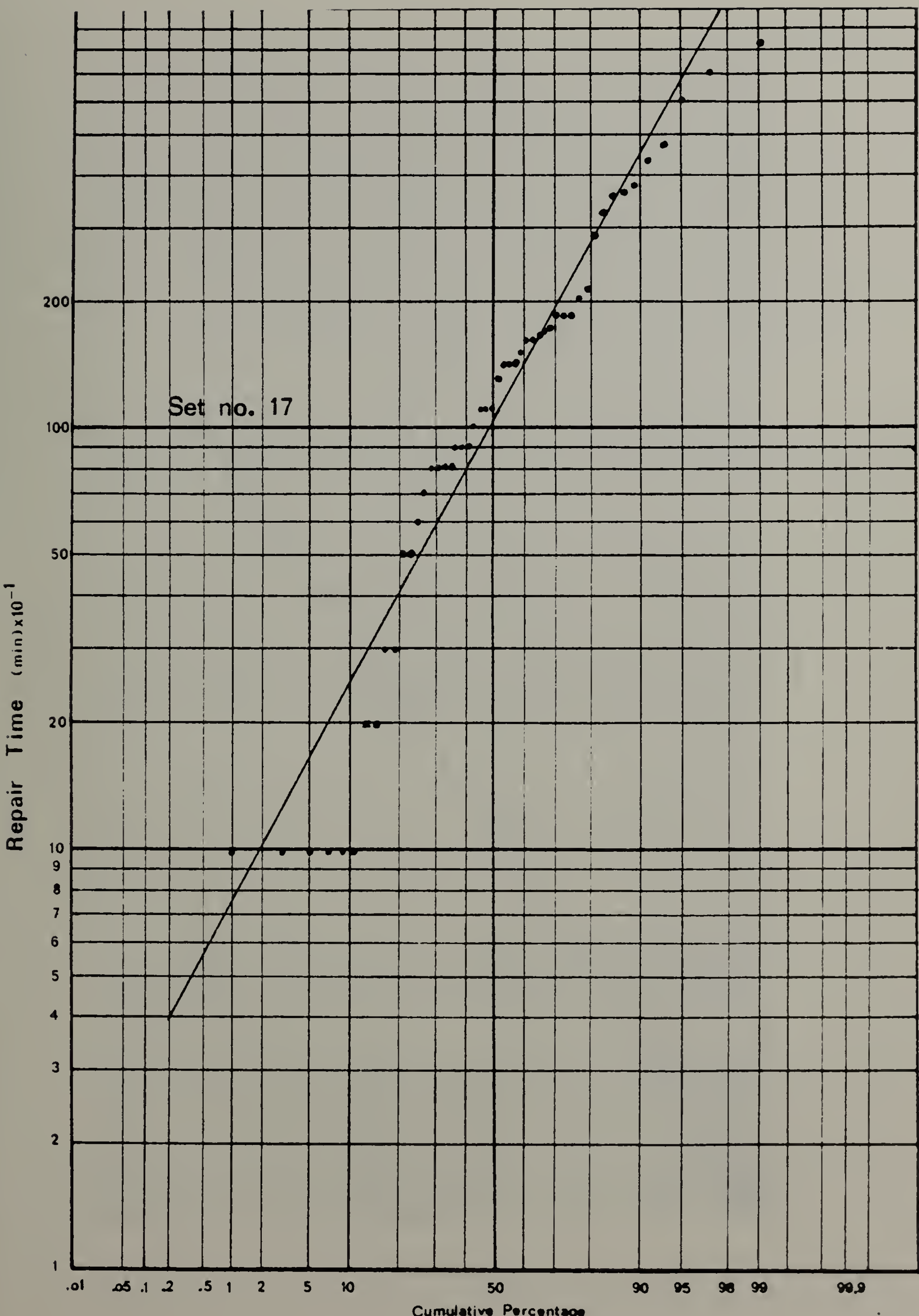
USASATCOMA HT/MT TERMINAL

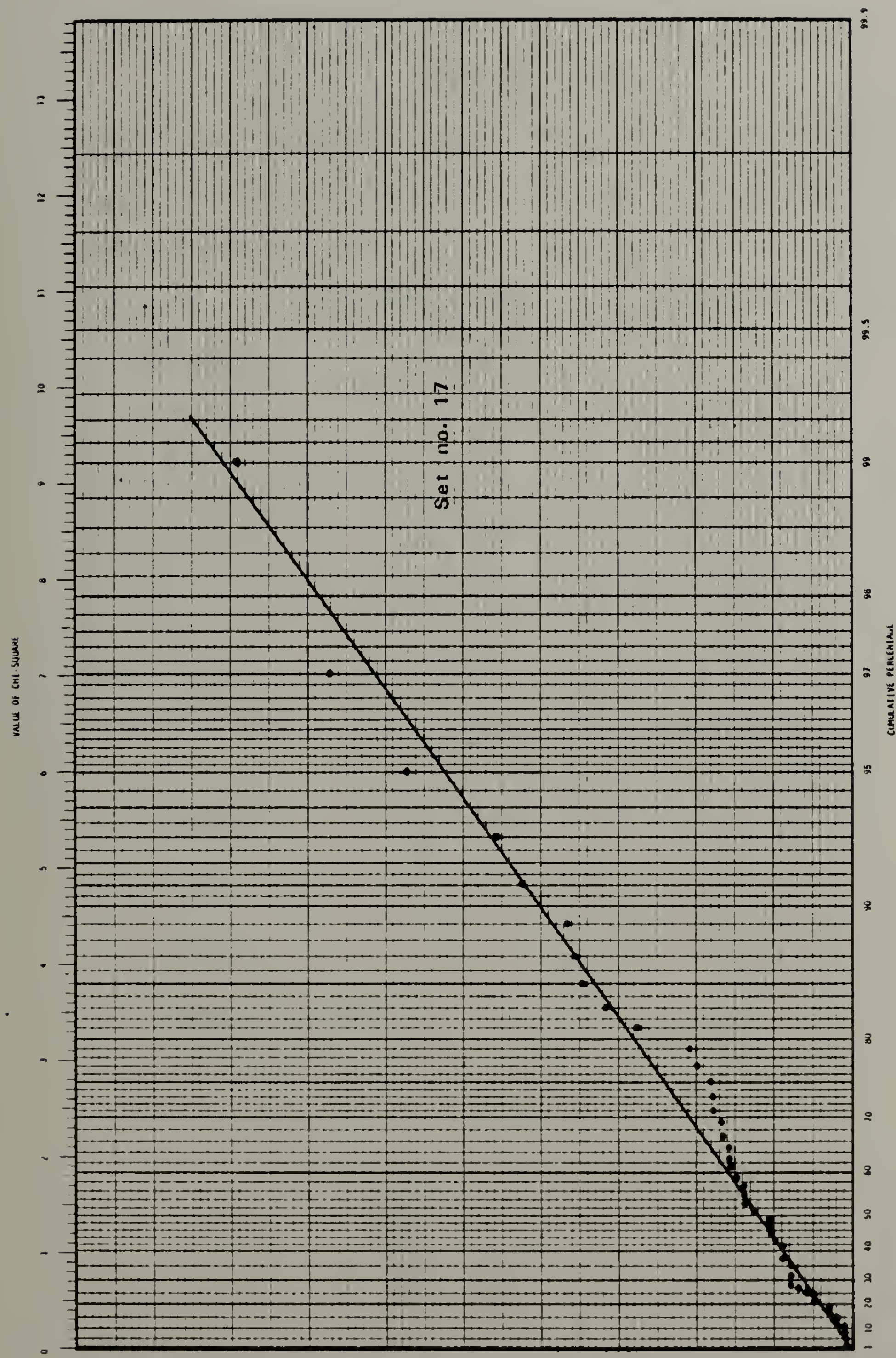
SAMPLE SIZE N = 50   NO. OF CELLS K = 10

SAMPLE MEAN = 17.02   STANDARD DEV = 17.40

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.06	2.29	
PARAM2		1.39	
MTTR	17.02	19.87	14.32 %
50-TH PERCNT	11.80	9.90	19.16 %
90-TH PERCNT	39.19	44.95	12.81 %
95-TH PERCNT	50.99	68.99	26.09 %
CHI-SQR STAT	8.40	20.00	
DEG OF FREED	8	7	
SIGNIF LEVEL	0.395E 00	0.557E-02	

W-TEST	
b^2	61.3
s^2	68.6
W STAT	0.894
SIGNIF LEVEL	< 0.01


Set no. 17
Repair Time (min)x10⁻¹
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage


Set no. 17
VALUE OF CHI-SQUARE

Set No. 18a

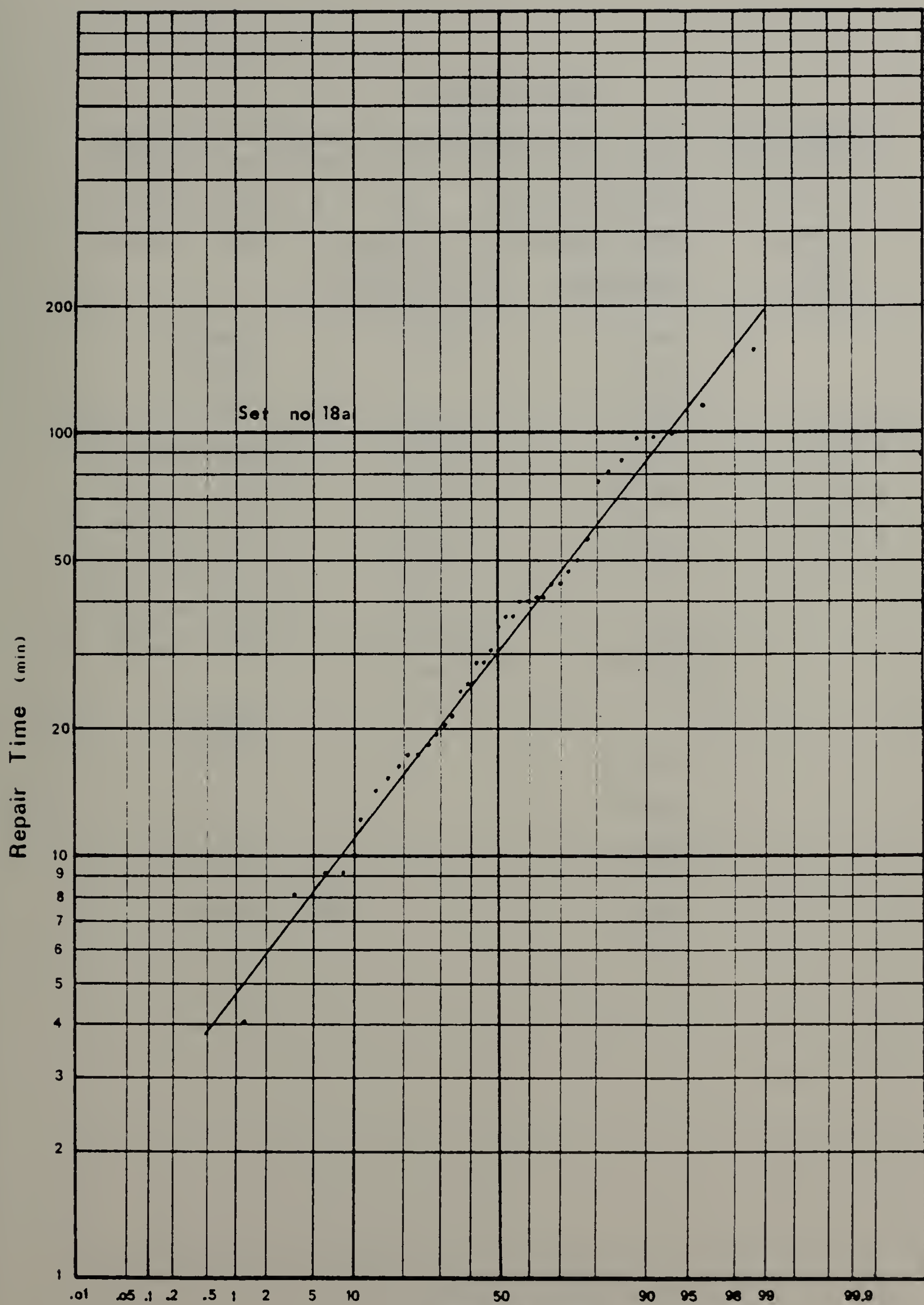
NATIONAL MILCOM. SYS.-GROUND DATA SYS. (INHERENT)

SAMPLE SIZE N = 39 NO. OF CELLS K = 7

SAMPLE MEAN = 41.64 STANDARD DEV = 33.60

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.02	3.43	
PARAM2		0.66	
MTTR	41.64	42.88	2.88 %
50-TH PERCNT	28.86	30.79	6.24 %
90-TH PERCNT	95.88	87.40	9.70 %
95-TH PERCNT	124.75	117.45	6.21 %
CHI-SQR STAT	9.64	9.64	
DEG OF FREED	5	4	
SIGNIF LEVEL	0.861E-01	0.469E-01	

W-TEST	
b^2	24.75
S^2	25.19
W STAT	0.982
SIGNIF LEVEL	0.85


Set no 18a
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 18b

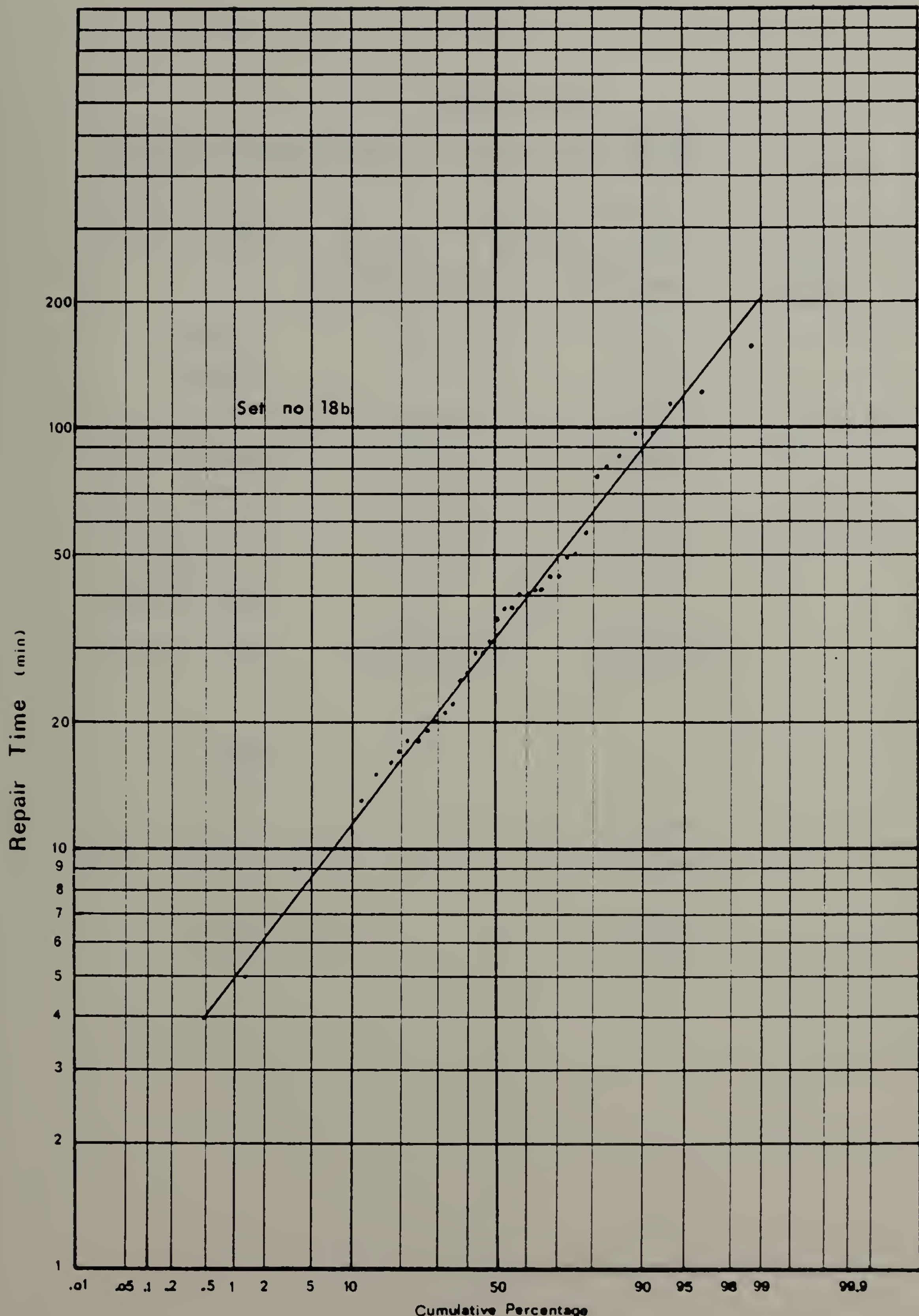
NATIONAL MILCOM. SYS.-GROUND DATA SYS. (ACHIEVED)

SAMPLE SIZE N = 39 NO. OF CELLS K = 7

SAMPLE MEAN = 43.28 STANDARD DEV = 34.73

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.02	3.48	
PARAM2		0.62	
MTTR	43.28	44.15	1.96 %
50-TH PERCNT	30.00	32.37	7.33 %
90-TH PERCNT	99.66	88.86	12.15 %
95-TH PERCNT	129.66	118.27	9.63 %
CHI-SQR STAT	11.08	6.77	
DEG OF FREED	5	4	
SIGNIF LEVEL	0.499E-01	0.149E 00	

W-TEST	
b^2	23.15
s^2	23.56
W STAT	0.983
SIGNIF LEVEL	0.86


Set no 18b
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01
.05
.1
.2
.5
1
2
5
10
50
90
95
98
99
99.9
Cumulative Percentage

Set No. 19

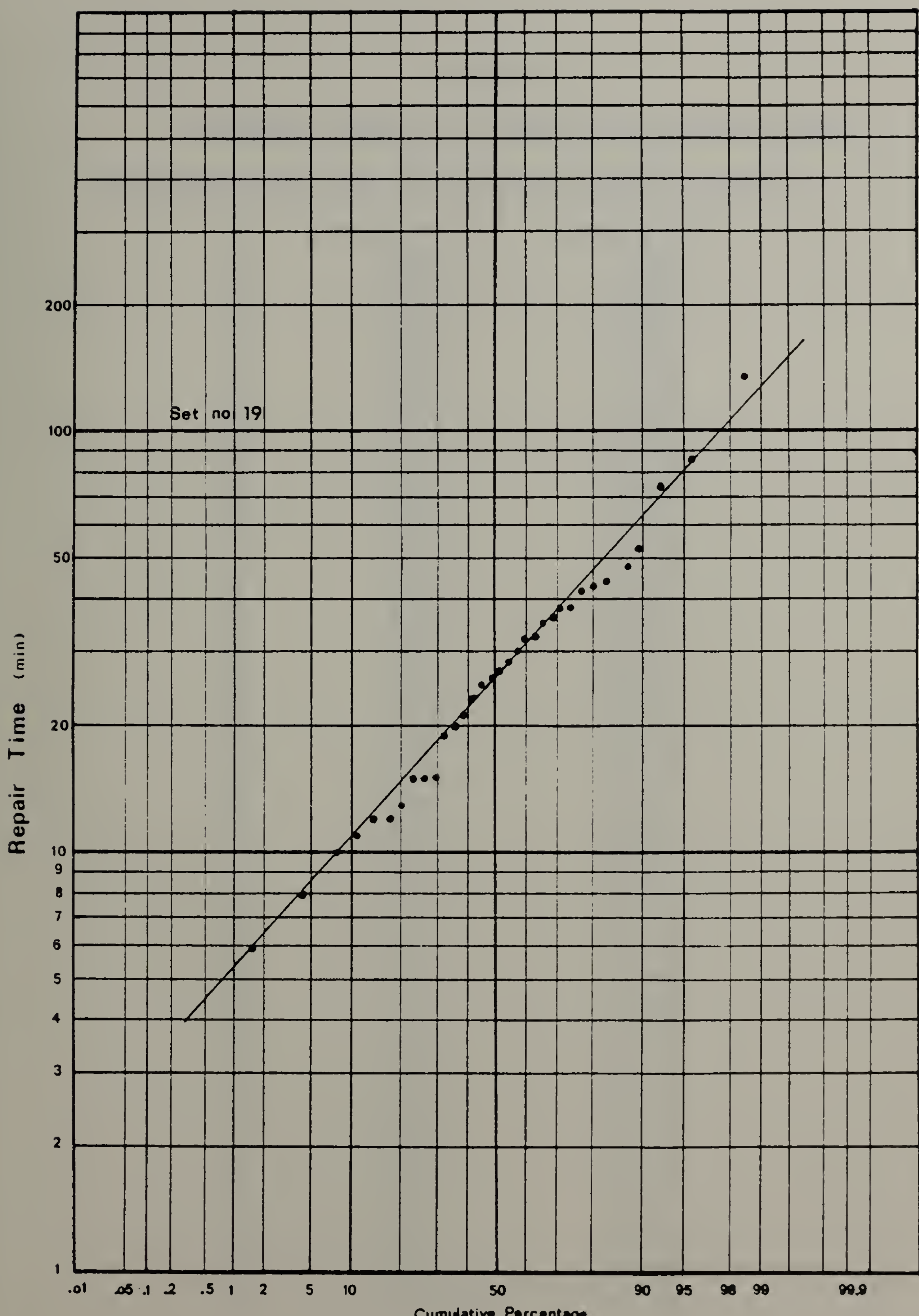
AUTODIN MEMORY/MEMORY CONTROL EQUIPMENT

SAMPLE SIZE N = 33 NO. OF CELLS K = 6

SAMPLE MEAN = 32.39 STANDARD DEV = 25.48

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.03	3.24	
PARAM2		0.48	
MTTR	32.39	32.52	0.39 %
50-TH PERCNT	22.45	25.57	12.18 %
90-TH PERCNT	74.59	62.22	19.89 %
95-TH PERCNT	97.04	80.03	21.26 %
CHI-SQR STAT	10.45	3.18	
DEG OF FREED	4	3	
SIGNIF LEVEL	0.334E-01	0.364E 00	

W-TEST	
b^2	15.21
s^2	15.39
W STAT	0.988
SIGNIF LEVEL	0.977



Set no 19
Repair Time (min)
200
100
50
20
10
9
8
7
6
5
4
3
2
1
.01 .05 .1 .2 .5 1 2 5 10 50 90 95 98 99 99.9
Cumulative Percentage

APPENDIX E

COMPLETE EXAMPLE OF COMPUTER PROGRAM OUTPUT

AN/GSA-51 BACK UP INTERCEPTOR CONTROL SYSTEM

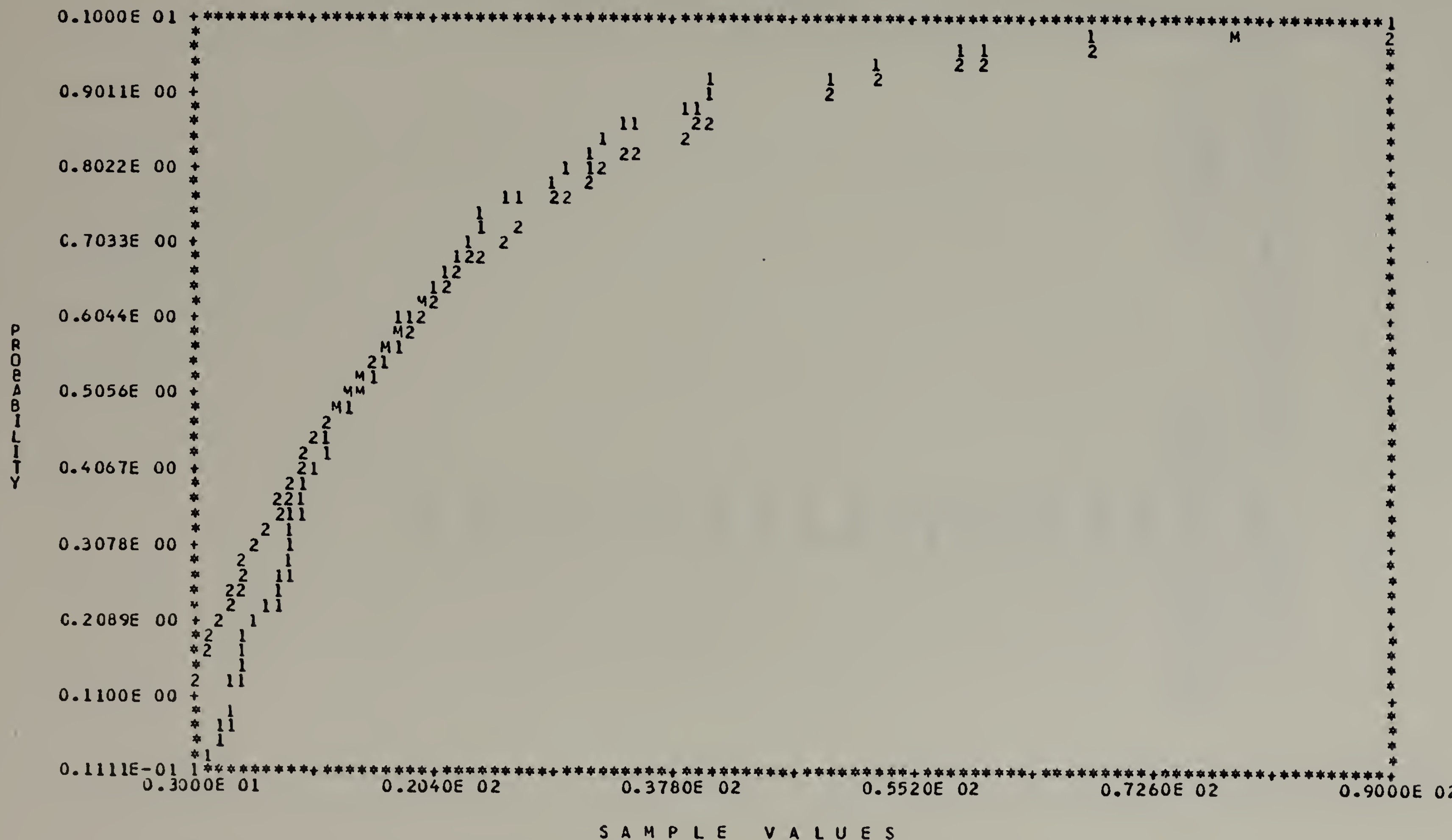
I	REPAIR TIME	APPROX F(I)
1	3.00	0.56
2	3.70	1.67
3	4.20	2.78
4	4.70	3.89
5	5.00	5.00
6	5.00	6.11
7	5.50	7.22
8	5.60	8.33
9	5.75	9.44
10	5.80	10.56
11	5.80	11.67
12	6.10	12.78
13	6.30	13.89
14	6.30	15.00
15	6.50	16.11
16	6.50	17.22
17	6.75	18.33
18	7.60	19.44
19	8.20	20.56
20	8.20	21.67
21	8.70	22.78
22	8.75	23.89
23	9.50	25.00
24	9.50	26.11
25	9.60	27.22
26	9.90	28.33
27	10.00	29.44
28	10.00	30.56
29	10.10	31.67
30	10.20	32.78
31	10.20	33.89
32	10.60	35.00
33	10.80	36.11
34	11.10	37.22
35	12.00	38.33
36	12.00	39.44
37	12.30	40.56
38	12.30	41.67
39	12.70	42.78
40	12.70	43.89
41	13.00	45.00
42	13.50	46.11
43	13.50	47.22
44	14.00	48.33
45	14.30	49.44
46	14.80	50.56
47	15.30	51.67
48	16.30	52.78
49	16.75	53.89
50	16.90	55.00
51	17.80	56.11
52	18.00	57.22
53	18.00	58.33
54	18.10	59.44
55	18.30	60.56

I	REPAIR TIME	APPROX F(I)
56	19.30	61.67
57	19.50	62.78
58	20.30	63.89
59	21.00	65.00
60	21.20	66.11
61	21.80	67.22
62	22.10	68.33
63	23.00	69.44
64	23.30	70.56
65	23.75	71.67
66	24.00	72.78
67	24.10	73.89
68	25.50	75.00
69	26.25	76.11
70	29.40	77.22
71	29.50	78.33
72	30.00	79.44
73	31.30	80.56
74	31.50	81.67
75	32.30	82.78
76	32.50	83.89
77	34.50	85.00
78	35.40	86.11
79	38.70	87.22
80	39.60	88.33
81	40.00	89.44
82	40.10	90.56
83	49.00	91.67
84	52.20	92.78
85	53.00	93.89
86	59.00	95.00
87	60.60	96.11
88	68.20	97.22
89	78.90	98.33
90	90.00	99.44

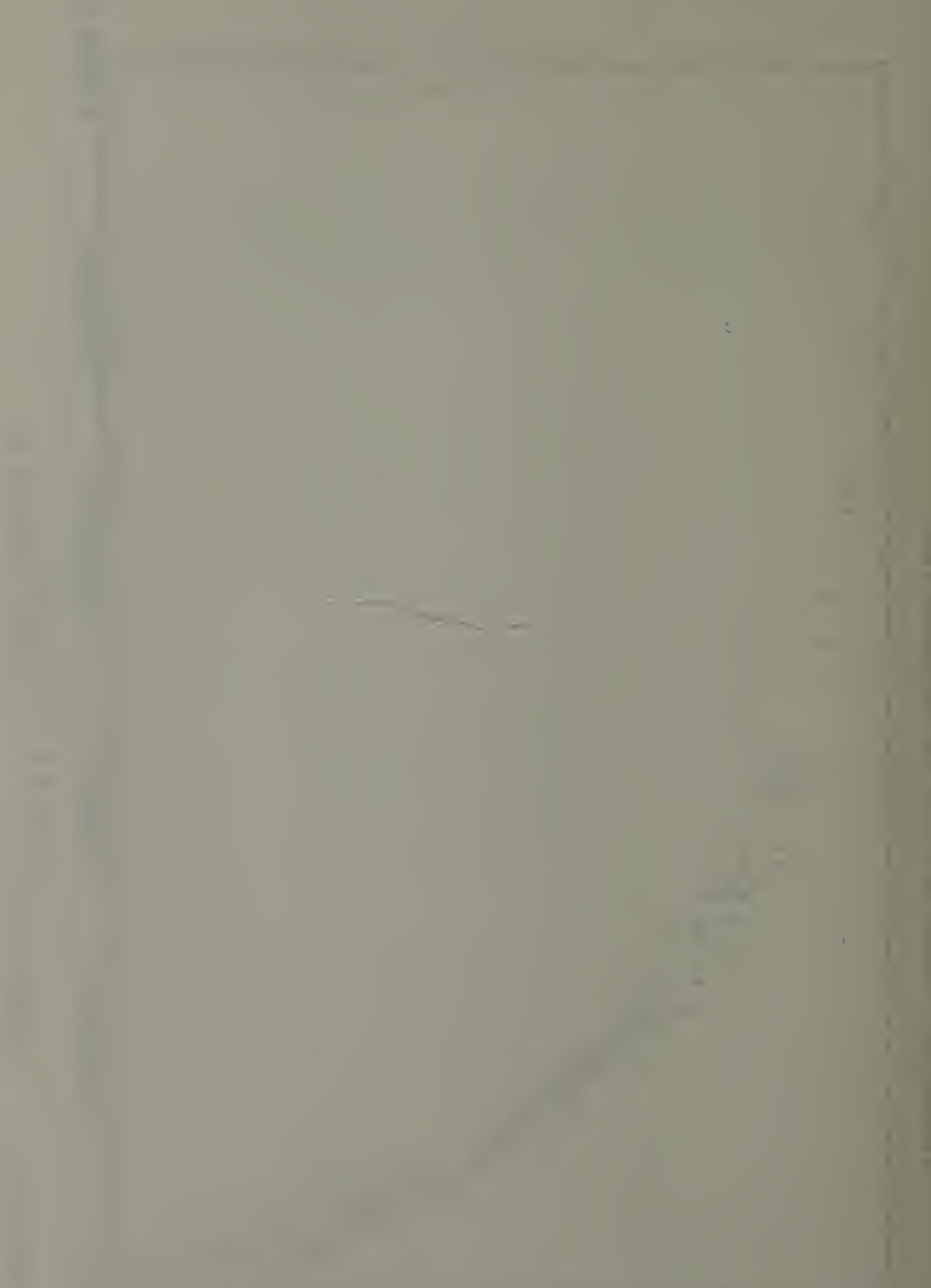
CHI-SQUARE COMPUTATION FOR : EXPONENTIAL

NO. OBS/CELL	CHI-SQUARE STATISTIC
0	5.00
0	5.00
2	1.80
4	0.20
10	5.00
4	0.20
8	1.80
8	1.80
8	1.80
4	0.20
7	0.80
7	0.80
6	0.20
4	0.20
6	0.20
4	0.20
4	0.20
4	0.20

EXPONENTIAL CUMULATIVE SAMPLE AND THEORETICAL PCFS



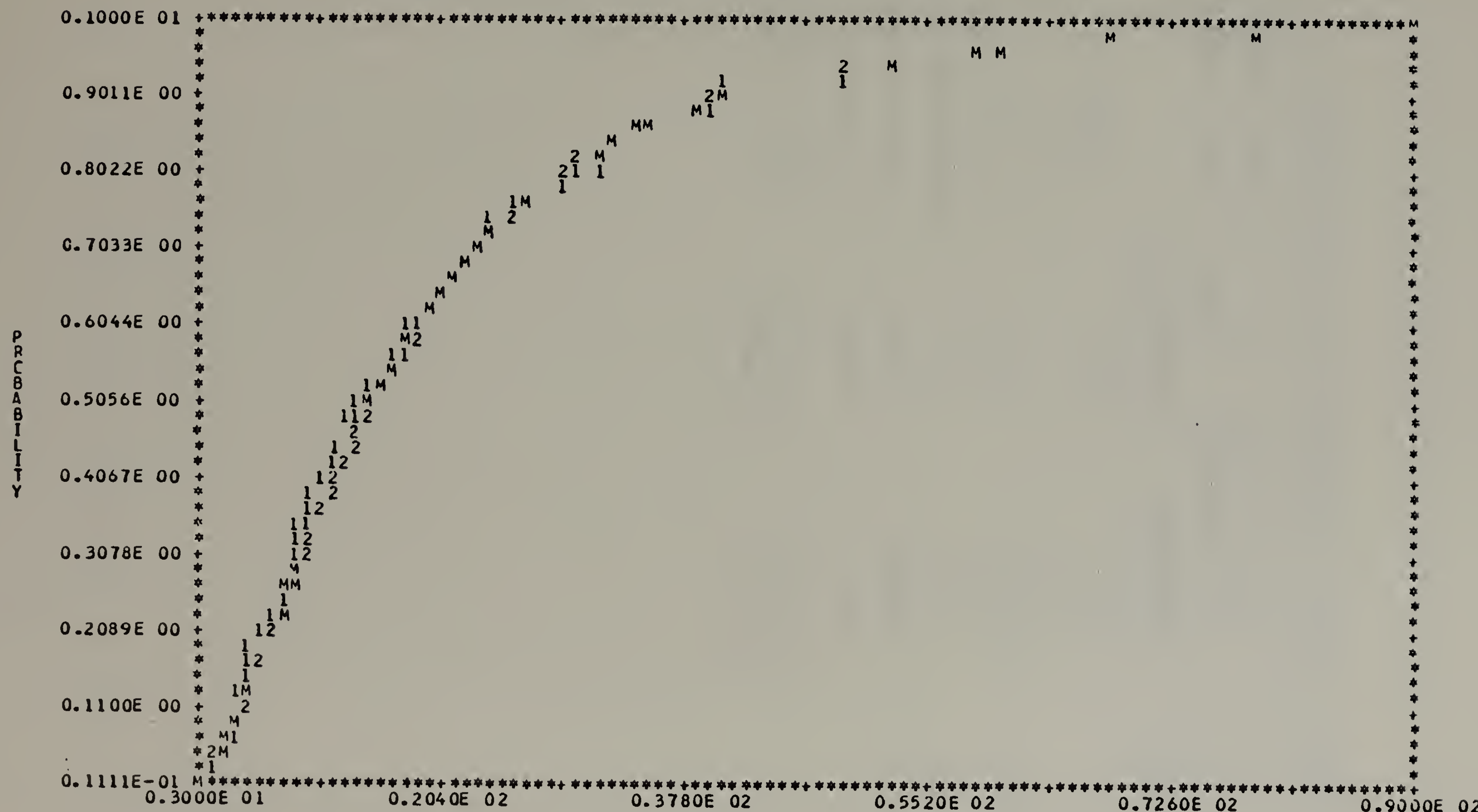
SAMPLE PDF = 1 THEORETICAL PCF = 2 MULTIPLE POINTS = M



CHI-SQUARE COMPUTATION FOR : LOGNORMAL

NO. OBS/CELL	CHI-SQUARE STATISTIC
3	0.80
8	1.80
6	0.20
3	0.80
5	0.00
8	1.80
5	0.00
5	0.00
4	0.20
3	0.80
5	0.00
5	0.00
5	0.00
4	0.20
5	0.00
4	0.20
5	0.00
7	0.80

LOGNORMAL CUMULATIVE SAMPLE AND THEORETICAL PDFS



S A M P L E   V A L U E S

SAMPLE PDF = 1   THEORETICAL PDF = 2   MULTIPLE POINTS = M

AN/GSA-51 BACK UP INTERCEPTOR CONTROL SYSTEM

SAMPLE SIZE N = 90 NO. OF CELLS K = 18

SAMPLE MEAN = 20.43 STANDARD DEV = 17.07

	EXPONENTIAL	LOGNORMAL	ERROR
PARAM1	0.05	2.73	
PARAM2		0.57	
MTTR	20.43	20.42	0.06 %
50-TH PERCNT	14.16	15.33	7.64 %
90-TH PERCNT	47.04	40.46	16.28 %
95-TH PERCNT	61.21	53.25	14.94 %
CHI-SQR STAT	25.60	7.60	
DEG OF FREED	16	15	
SIGNIF LEVEL	0.599E-01	0.939E 00	

APPENDIX F

COMPUTER PROGRAM FOR DATA ANALYSIS

The program makes use of routines from the International Mathematical and Statistical Library (IMSL), which NPS Computer Center users are authorized to use.

The description of the function and parameters of the routines is given in the IMSL user's manual which is available at NPS Computer Center [Ref. 29].

The following is a list of the routines from the IMSL used in the program:

1. VSORTA - Sort arrays in ascending order.
2. GFIT - Chi-squared goodness-of-fit test.
   a. MCDFI - Chi-square probability distribution function (PDF).
3. USPC - Print and plot sample and theoretical PDF with 95% band confidence intervals.
   a. VSMMMX- Locates the Min and Max values of a vector.
   b. USPLH - Printer Plot.

In addition, two External PDF's are defined:

1. REXP - PDF of the Exponential distribution.
2. RLOG - PDF of the Lognormal distribution.

The main program computes the following:

1. Expected values of ordered observations ($\frac{i-0.5}{N}$ x 100).
2. Sample mean and standard deviation based on the maximum likelihood estimates.
3. The parameters of the lognormal distribution (μ,σ^2).

4. Calculated 50th, 90th, and 95th percentiles under exponential and lognormal distributions assumptions.

5. Percentage differences between the calculated parameters of the exponential and lognormal distributions.

Input Formats

Card Type #	Variable Name	Cols.	Format	Description
1	NS	1-2	I2	Number of data sets.
2	Title	1-50	50A1	Name of system/equipment analyzed, or any other identification. The content of this card is printed out.
3	IN	1-3	I3	Number of observations in the data set analyzed.
	IK	4-6	I3	Number of equiprobable cells in the data set analyzed.
4	Data(I) I=1,IN	1-80	8F10.5	Each card contains 8 observations. Only the last card can be less than 8, depending on the number of observations in the data set. The maximum number of cards of this type, for each data set, is 999.

Cards No. 2, 3, and 4 are repeated in this order for each data set.

```
      DIMENSION DATA(240),DCELLS(60),OCOMP(60),WA(2248,1),
     1      TITLE(50),F(240),NCEL(60),DATX(240)
      COMMON IN,IK,PMEAN,VAR,RLMEAN,RLVAR,FPE,FPL
      EXTERNAL REXP,RLOG

** READ IN INPUT DATA **
      READ (5,10) NS
   10 FORMAT (I2)
       DO  999  L = 1,NS
      READ (5,15)  (TITLE(K), K = 1,50  )
   15 FORMAT (50A1)
      WRITE (6,99)
      WRITE (6,20) (TITLE(K), K = 1,50  )
   20 FORMAT ('0',15X,50A1)
      WRITE (6,22)
   22 FORMAT (' ',15X,50('-'))
      READ  (5,30)  IN, IK
   30 FORMAT  (I3,I3)
      READ  (5,40)  (DATA(I),I=1,IN)
   40 FORMAT (8F10.5)

      CALL   VSORTA  (DATA,IN )

** COMPUTE AND PRINT 'EXPECTED VALUES
   OF ORDERED OBSERVATIONS' **
      J60 = 55
      WRITE (6,45)
   45 FORMAT ('0',26X,'I',5X,'REPAIR TIME',5X,'APPROX F(I)')
      DO  50 J = 1,IN
      FJ = J
      F(J) = (FJ - 0.5 )/ IN * 100
      WRITE (6,48) J,DATA(J),F(J)
   48 FORMAT (' ',25X,I3,5X,F8.2,7X,F8.2)
      IF ( J .LT. J60 ) GO TO 50
      WRITE (6,99)
      WRITE (6,45)
      J60 = J60 + 55
   50 CONTINUE

** COMPUTE DISTRIBUTIONS PARAMATERS **
      SUM = 0.0
      SUMS = 0.0
      PMEAN = 0.0
      VAR  =  0.0
      IN1 = IN - 1
      DO  60  J = 1,IN
      SUM = SUM + DATA(J)
      SUMS = SUMS + DATA(J)**2
   60 CONTINUE
      PMEAN = SUM / IN
      VAR = (SUMS/IN1) - (SUM**2)/(IN*IN1)
      STDV  = SQRT( VAR )
      PMEANE = 1/PMEAN
      VARE  =  1/PMEAN
      SML = 0.0
      SMLS= 0.0
      RLMEAN = 0.0
      RLVAR = 0.0
      DO  500  K = 1,IN
      SML = SML + ALOG(DATA(K))
      SMLS = SMLS + ( ALOG(DATA(K)))** 2
  500 CONTINUE
      IN1 = IN - 1
      RLMEAN = SML / IN
      RLVAR =  (SMLS/IN1 ) - (SML ** 2) / (IN * IN1 )
      PMEANL  =  RLMEAN
      PVARL = RLVAR
      STDVL = SQRT(PVARL)
      CMEANL = EXP(PMEANL + 0.5*PVARL)
```

```
** COMPUTE PERCENTAGE ERROR BETWEEN EXP AND LOGN PARAM.
      PER50  =  EXP(PMEANL)
      PER90  =  EXP(PMEANL + 1.282 * STDVL )
      PER95  =  EXP(PMEANL + 1.645 * STDVL )
      PERE50  = -ALOG(0.50) * PMEAN
      PERE90  = -ALOG(0.10) * PMEAN
      PERE95  = -ALOG(0.05) * PMEAN
      EMEAN =  ABS(CMEANL - PMEAN)/CMEANL       * 100
      EPER50  = ABS (PER50 - PERE50 ) / PER50   * 100
      EPER90  = ABS (PER90 - PERE90 ) / PER90   * 100
      EPER95  = ABS (PER95 - PERE95 ) / PER95   * 100

      MN1 = 1
      MN9 = 95
      MIP1 = 1
      MIC1 = 0
      IDFE = 1
      IDFL = 2

** PERFORM CHI-SQUARE TEST FOR EXP ASSUMPTION **
      CALL  GFIT (REXP,IK,DATA,IN,DCELLS,DCOMP,DCSE,IDFE,
     1          XQE,IER)
      DO   600  K = 1,IK
      NCEL(K) = DCELLS(K)
  600 CONTINUE
      WRITE (6,99)
      WRITE (6,610)
  610 FORMAT ('0',20X,'CHI-SQUARE COMPUTATION FOR :',
     1        ' EXPONENTIAL ')
      WRITE (6,615)
  615 FORMAT (' ',20X,40('-'))
       WRITE (6,620)
  620 FORMAT ('0',23X,'NO. OBS/CELL',7X,'CHI-SQUARE ',
     1        'STATISTIC')
      WRITE (6,625)
  625 FORMAT (' ',23X,12('-'),7X,20('-'))
      WRITE (6,630) ( NCEL(K),DCOMP(K),K = 1,IK )
  630 FORMAT ('0',28X,I2,16X,F6.2)
      DO   640 I = 1,IN
      DATX(I) = DATA(I)
  640 CONTINUE

** PLOT OF EXP THEORETICAL AND SAMPLE PDF **
      CALL  USPC  (REXP,DATX,IN,MN1,MN9,MIP1,MIC1,WA)

** PERFORM CHI-SQUARE TEST FOR LOGN ASSUMPTION **
      CALL  GFIT (RLOG,IK,DATA,IN,DCELLS,DCOMP,DCSL,IDFL,
     1        XQL,IER)
      DO  650  K = 1,IK
      NCEL(K) = DCELLS(K)
  650 CONTINUE
      WRITE (6,99)
      WRITE ( 6,655)
  655 FORMAT ('0',20X,'CHI-SQUARE COMPUTATION FOR : ',
     1        ' LOGNORMAL ')
      WRITE ( 6,615)
      WRITE (6,620)
      WRITE ( 6,625)
      WRITE (6,630)   (NCEL(M),DCOMP(M), M = 1,IK)
      DO  660 I = 1,IN
      DATX(I) = DATA(I)
  660 CONTINUE
```

```
** PLOT OF LOGN THEORETICAL AND SAMPLE PDF **
      CALL  USPC  (RLOG,DATX,IN,MN1,MN9,MIP1,MIC1,WA)

** PRINT SUMMARY TABLE OF RESULTS **
      WRITE (6,100) (TITLE(K),K = 1,50 )
  100 FORMAT ('0',25X,50A1)
      WRITE (6,102)
  102 FORMAT (' ',25X,50('-'))
      WRITE (6,104) IN,IK
  104 FORMAT ('0',25X,'SAMPLE SIZE  N = ',I3,5X,'NO. OF',
     1       ' CELLS  K = ',I2)
      WRITE (6,106) PMEAN,STDV
  106 FORMAT ('0',25X,'SAMPLE MEAN = ',F6.2,5X,'STANDARD ',
     1     'DEV = ',F6.2)
      WRITE (6,110)
  110 FORMAT ('0',39X,'EXPONENTIAL',4X,'LOGNORMAL',5X,
     1         'ERROR')
      WRITE (6,115)
  115 FORMAT (' ',39X,11('-'),4X,9('-'),5X,5('-'))
      WRITE (6,125) PMEANE,PMEANL
  125 FORMAT ('0',29X,'PARAM1',7X,F6.2,8X,F6.2)
      WRITE (6,130)  PVARL
  130 FORMAT ('0',29X,'PARAM2',21X,F6.2)
      WRITE (6,135) PMEAN,CMEANL,EMEAN
  135 FORMAT ('0',30X,'MTTR',8X,F6.2,8X,F6.2,6X,F5.2,' %')
      WRITE (6,140) PERE50,PER50,EPER50
  140 FORMAT ('0',25X,'50-TH PERCNT',5X,F6.2,8X,F6.2,6X,
     1      F5.2,' %')
      WRITE (6,150) PERE90,PER90,EPER90
  150 FORMAT ('0',25X,'90-TH PERCNT',5X,F6.2,8X,F6.2,6X,
     1      F5.2,' %')
      WRITE (6,160) PERE95,PER95,EPER95
  160 FORMAT ('0',25X,'95-TH PERCNT',5X,F6.2,8X,F6.2,6X,
     1      F5.2,' %')
      WRITE (6,200) DCSE,DCSL
  200 FORMAT ('0',25X,'CHI-SQR STAT',5X,F6.2,8X,F6.2)
      WRITE (6,210) IDFE,IDFL
  210 FORMAT ('0',25X,'DEG OF FREED',7X,I2,12X,I2)
      WRITE (6,220) XQE,XQL
  220 FORMAT ('0',25X,'SIGNIF LEVEL',3X,D11.3,3X,D11.3)
      WRITE (6,225)
  225 FORMAT (' ',42X,9('-'),5X,9('-'))
  999 CONTINUE
      WRITE  (6,99)
   99 FORMAT ('1')
      RETURN
      END


      SUBROUTINE  REXP  ( XE,PE )
      COMMON IN,IK,PMEAN,VAR,RLMEAN,RLVAR,FPE,FPL
      PE = 1 - EXP (-XE/PMEAN)
      RETURN
      END


      SUBROUTINE  RLOG  ( XL,PL )
      COMMON IN,IK,PMEAN,VAR,RLMEAN,RLVAR,FPE,FPL
      TX = ( ALOG(XL) - RLMEAN) / SQRT(RLVAR)
      TX1 = TX * 0.7071068
      PL = 0.5 + 0.5 * ERF(TX1)
      RETURN
      END
```

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A407 Almog
c.1
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